Helical Tiling – Columns built with flat Tiles

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Abstract

In general a tiling is considered to be a set of tiles placed next to each other on a flat surface. The tiles are placed on the surface in such a way that there are no gaps and no overlaps. We consider the case in which the surface is not flat For when there are no gaps and no overlaps between the tiles we still can call it a tiling. The consequences for the possible shapes of the tiles in non-flat tilings as the possibility to build columns with one type of tile only are discussed in this paper.

1. Tilings

1.1. Definition. A tiling, or tessellation, is a covering of a plane without gaps or overlaps by polygons, all of which are the same size and shape. That is one of the definitions of a tiling. Another definition is the following: Tiling: a pattern made of identical shapes; the shapes must fit together without any gaps and the shapes should not overlap. Although the second definition doesn't speak about a plane, it is mostly assumed that the tiles do cover a plane. But we can take the definition literal, and then the only conditions are that the tiles do not overlap and do not leave gaps. Tilings in which all the tiles have the same shape are mostly called monohedral tilings [1]. All tilings in this paper use a single, planar, polygonal tile, and these tiles are connected edge-to-edge so that they form a water-tight 2-manifold; the edges can join with different dihedral angles. Thus the resulting manifold can be perfectly flat (Figure 1) or can have some 3D structure (Figure 2). Furthermore, the overall shape can have topology of the infinite plan (Figure 2) or have the connectivity of a cylinder of finite diameter (Figure 3) – forming structural columns.

Figure 1: *L-shaped tiles.*

Figure 2: *A non-flat tiling.*

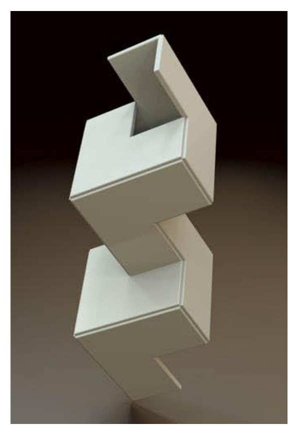


Figure 3: *Cylindrical tiling.*

1.2. Non-flat tilings. Besides the possibility shown in Figure 1, where the tiles are put together in such a way that they cover a flat plane, there are a few more ways to put the tiles together under the condition that we do not want overlaps or gaps in the construction. In Figure 2 the L-shaped tiles are set up in such a way that we can make a three dimensional construction. As you can see the structure is made out of identical shapes. The tiles do not overlap and there are no gaps in the structure. To call it a tiling in the traditional way, the only remark one can make is that it is not a covering of a flat surface. But it still is a tiling. Also when we combine the tiles in the way shown in Figure 3, the resulting structure is a tiling.

2. Monohedral Cylinders

2.1. Stacked Polyhedra. The inspiration of such columns may come from stacking regular polyhedra such as the Platonic solids (Figure 4). In doing so one would remove the faces by which the polyhedral are joined; the remaining faces then again constitute a 2-manifold in the form of a monohedral tiling (Figure 5).

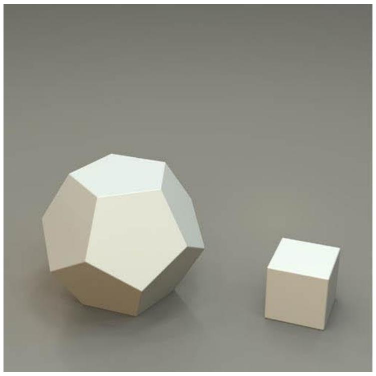


Figure 4: *Dodecahedron and cube.*

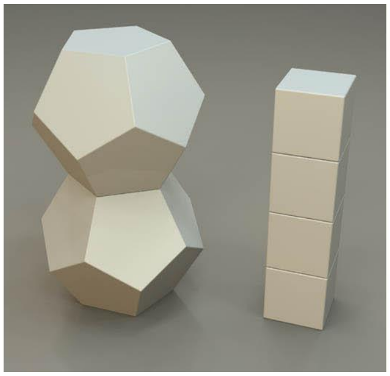


Figure 5: *Stacking polyhedra.*

Figure 6: *Compressing.*

Figure 6 shows that if regular polyhedra are joined while allowing them to intersect each other, they can still result in a regular tiling pattern on the resulting surface, if their respective orientation is carefully chosen. Note, that the cubes are now stacked along their space diagonals. In the case where we started with the cubes we end up with a column built with the same L-shaped tiles as in Figure 3. Two ways of using the L-shaped tiles to build a column are shown in Figure 7.

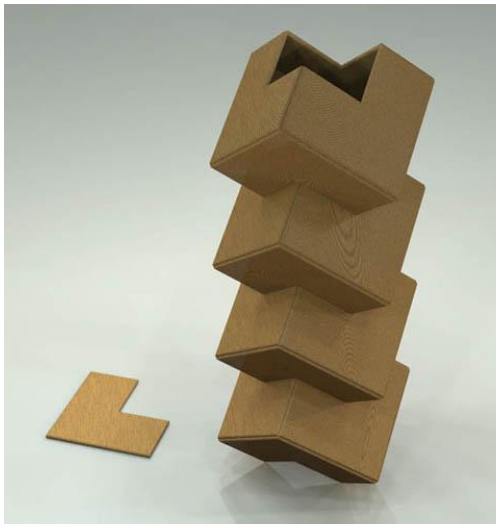


Figure 7: *Columns built with L-shaped tiles.*

3. Helical Columns

3.1. New shapes for cylindrical tilings. To create new and more interesting shapes of tiles for cylindrical tilings I developed the following method: First draw a normal helical curve. This curve is divided in equal parts. After that draw a straight line from the start point of the first part to the start point of the next part. And so on. Then extrude these straight lines downwards in a conical direction towards the axis of the spiral, as in Figure 8a. We need this basic shape that is needed to construct the shape of the tiles for the cylindrical tiling. Turn the shape of Figure 8a upside down and add it to the original shape (Figure 8b). Both shapes do intersect, and from the intersection lines we can derive the final shape of the tile (Figure 8c,d). We show the completed tiling in Figure 9a. The shape of the tile is a non-convex hexagon which will not tile a plane. New tilings, of which we can see several examples in Figure 9 and 10, are created this way. To develop the variety of shapes I used the 3D drawing program Rhinoceros and the programming tool Grasshopper. In the programme the radius and the pitch of the helix can be adjusted, as well as the distance between the division points. Finally the length of the extrusion can be chosen. Setting the values for these variables will deliver the final tile automatically. In Figure 9 and 10 only a few examples are shown.

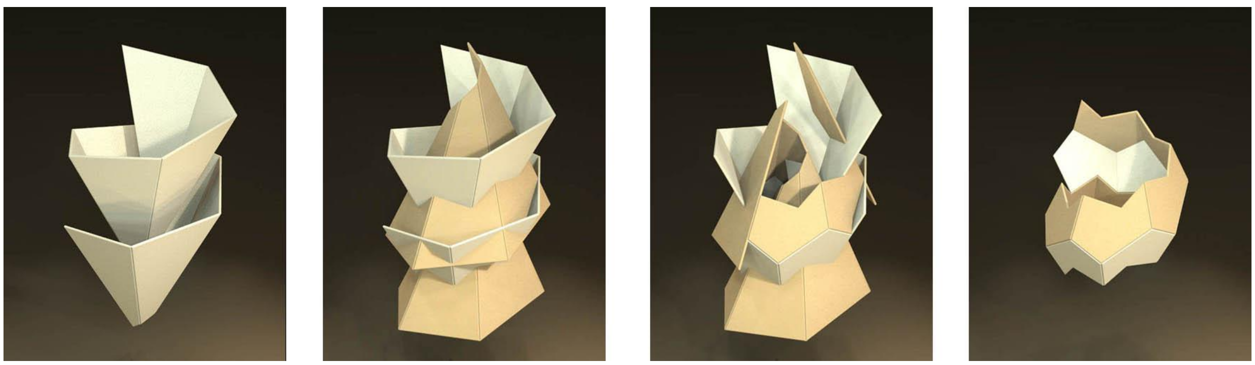


Figure 8a,b,c,d: *Developing the shape of the tile for the helical tiling of a column.*

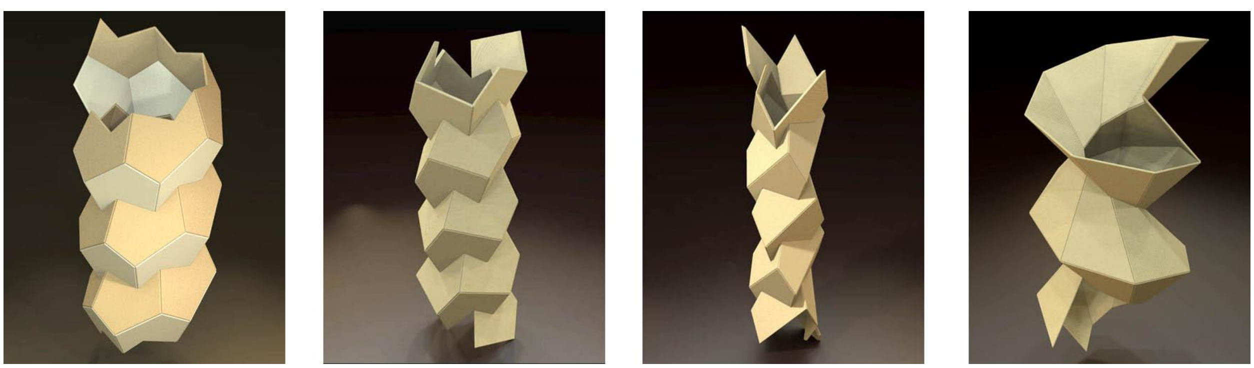


Figure 9a,b,c,d: *A few examples of the possible shape of the tiles derived from the Grasshopper definition.*

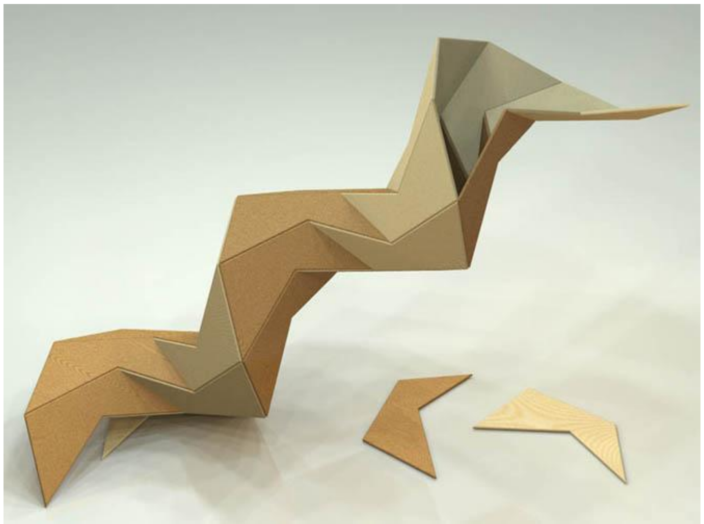


Figure 10: *Monohedral helical tiling.*

4. Sculptures

4.1. From virtual to real. For an assignment I designed a collection of seven sculptures which have to be placed aside a river in the Netherlands. I decided to use the Grasshopper program to develop the collection of the seven different tiles (Figure 11). I then built the models by laser cutting the pieces out of paper. To make the models you just cut a strip of tiles and roll it into the final shape.

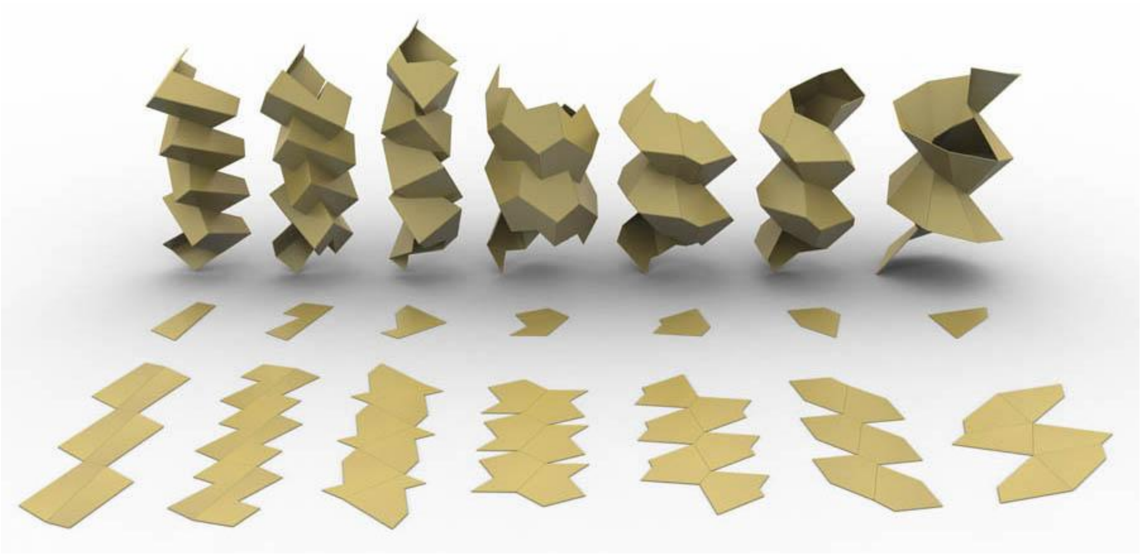


Figure 11: *A collection of seven columns which will be build in Corten steel.*

4.2. The steel sculptures. The next step is the making of the steel sculptures, of height about 3.3 meters. The parts, which are all equal, are cut out by laser cutting and welded together. Figure 12 shows the first sculpture placed near the water.

Figure 12: *One of the seven sculptures made in Corten steel.*

5. Equilateral Triangles as Tiles

5.1. Stacked Antiprisms. Another approach to develop helical colums with one shape of tile starts with stacking antiprisms (Figure 13).

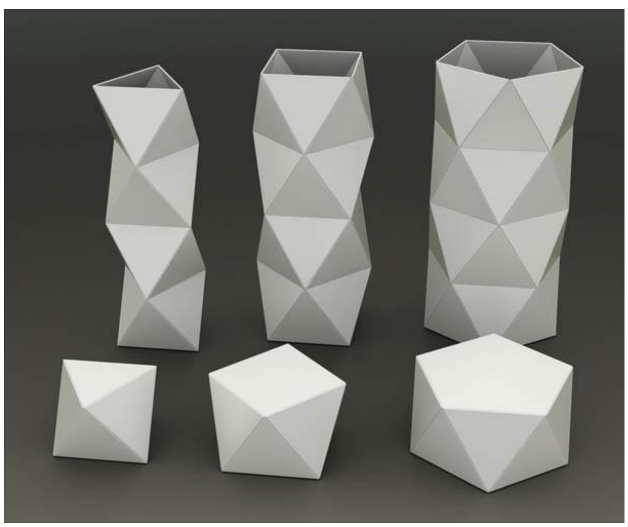
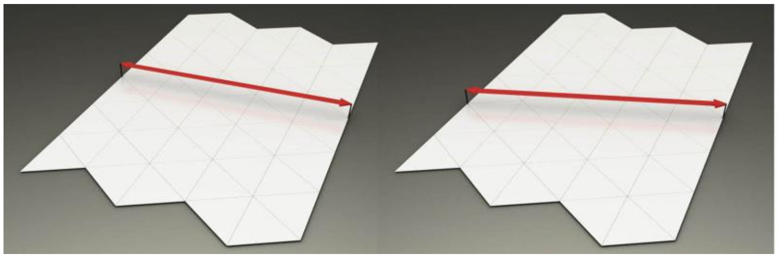


Figure 13: *Stacks of antiprisms.*

In their book "Infinite Polyhedra" [3], Wachman, Burt and Kleinmann show that another group of infinite cylindrical polyhedra can be made by the use of helicoidal strips of equilateral triangles. In fact you can create this structure by shifting the point of connection when you create the column from a sheet of paper folded in a pattern of equilateral triangles (Figure 14). Figure 15 shows the difference of the two ways of connecting.



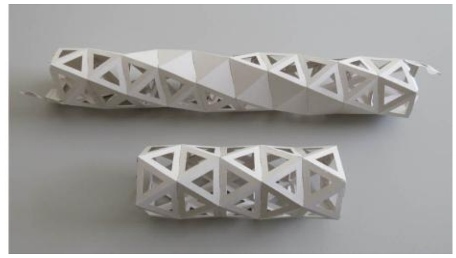
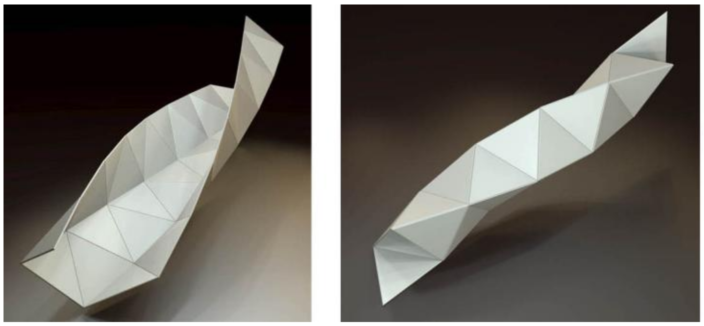


Figure 14: *Shifting the connection.* **Figure 15**: *Before and after shifting.*



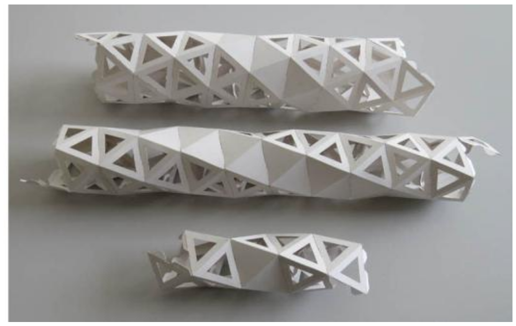
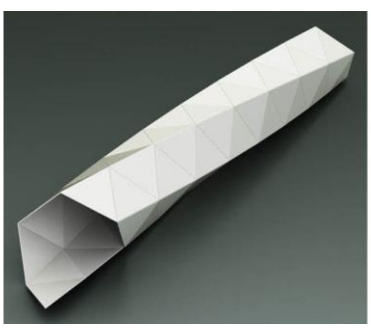


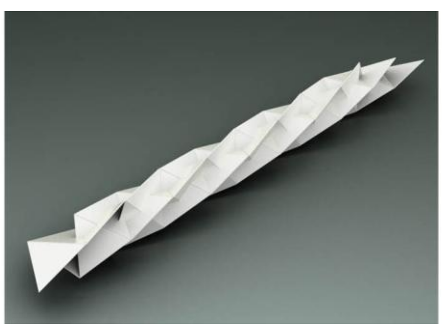
Figure 16: *Folding the tetrahelix.* **Figure 17**: *Helical deltahedra.*

5.2. Folding. So when we use folding sheets with equilateral triangles as the technique to create the helical columns, the thickness of the final column is determined by the number of parallel strips in the sheet.

6. New Uniform Polyhedra

6.1. The discovery of a new series of Uniform Polyhedra. We can extend the folding technique to allow intersecting. This idea was essential for my discovery of the new series of Uniform Polyhedra which I presented at the Bridges Conference in 2013 [4]. Instead of connecting the edges after folding the paper once (Figure 18), we can go on with the folding process (at least in the virtual world) until the edges meet again (Figure 19 and 20), essentially winding twice around the helix axis.





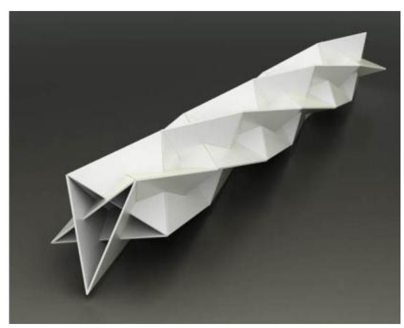


Figure 18: *Helical column.* **Figure 19**: *With intersecting.* **Figure 20**: *Uniform polyhedron.*

6.2. Making models. These so-called *Helical Star Deltahedra* have two properties in common: they have only regular faces (all faces are equilateral triangles) and all vertices are congruent (at each vertex six triangles are joined together). Therefore it is relative easy to build models of these object. The first way to do this is with double-triangle paper elements, made by laser cutting (Figure 21).

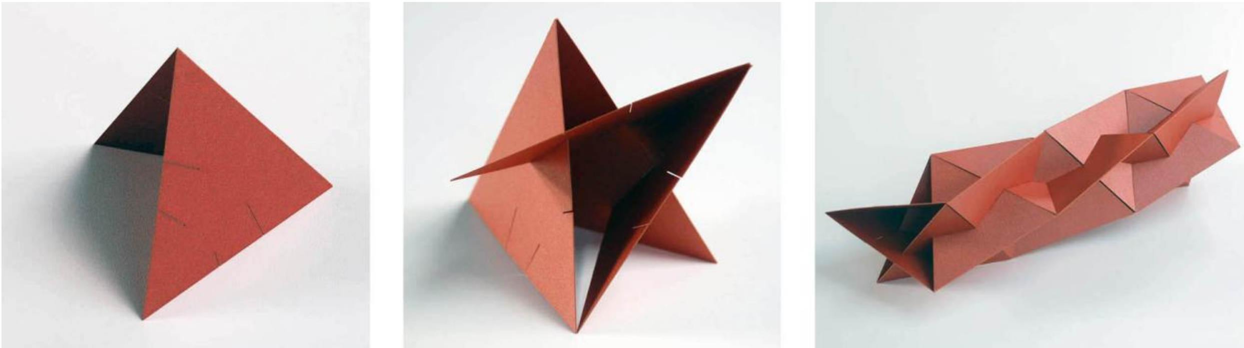


Figure 21: *Making the paper model.*

6.3. Many possibilities. Depending on the number of strips, the number of shifts and the number of times you fold before you connect, you can create all the different helical star deltahedra. The group has an infinite number of members. We show two more in Figure 22 and 23.

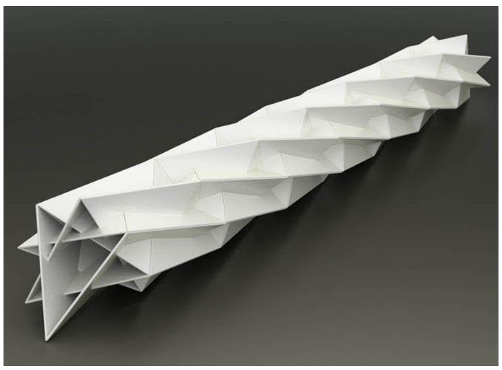


Figure 22: *Helical Star Deltahedron.*

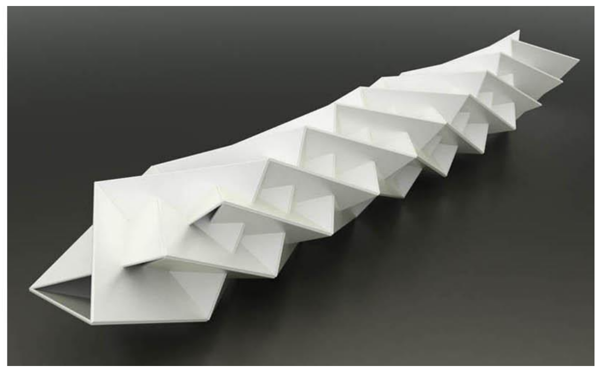


Figure 23: *Helical Star Deltahedron.*

7. Sculptures

7.1. Gathering for Gardner. After making the designs and the first paper models I was able to create bigger sculptures using this idea. I based the first one on the edge model of Figure 24. In this set up only one type of connector is present, and all pipes between the connectors have the same length. I made all connectors for the final sculpture with a 3D printer.

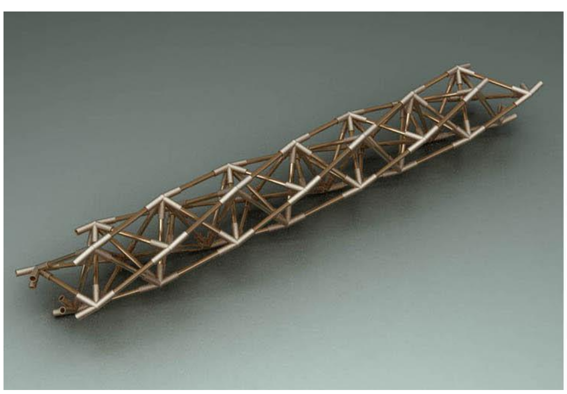


Figure 24: *Edge model.*

Figure 25: *Sculpture at G4G10.*

7.2. Elblag, Poland. To be able to make the steel sculpture, I first made a 3D printed model.

Figure 26: *Printed model.*

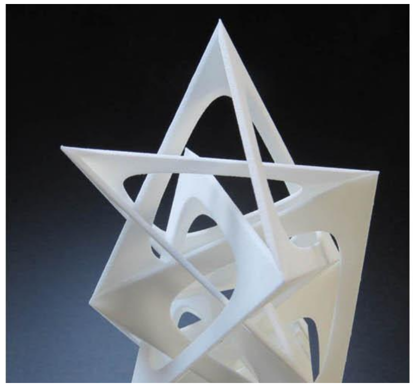


Figure 27: *Detail: star shape.*

Figure 28: *The Welded star.*

And with this model I could instruct the steel workers to assemble the final sculpture.

Figure 29: *Connected triangles.*

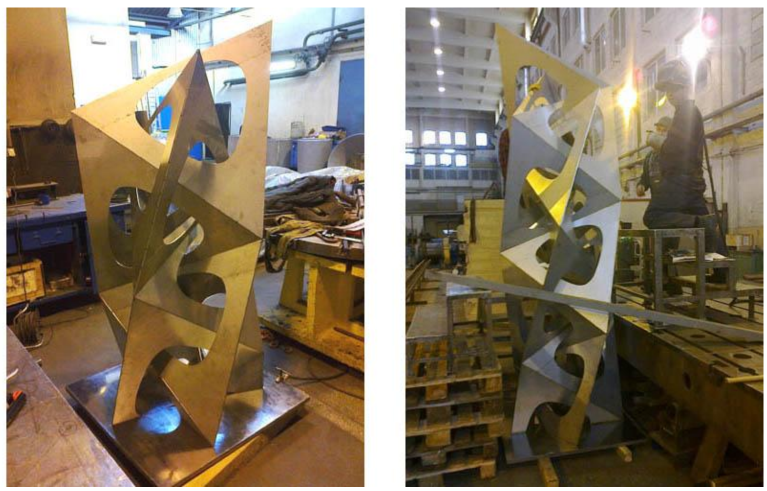


Figure 30: *The making of the sculpture for Elblag.*

Figure 31: *Sculpture in Elblag, Poland.*

7.3. Non-flat Tilings. I think the most fascinating aspect of this project is that we see that with the limitation of using one type of tile for a three dimensional construction the number of possibilities is still unlimited and it leads to interesting unexpected sculptures.

References

[1] Grünbaum and Sheppard, *Tilings and Patterns,* W.H. Freeman and Company, New York, 1987.
[2] Rinus Roelofs, *Non-flat tilings with flat tiles*, Bridges Proceedings, Banff, 2009.
[3] Wachman, Burt and Kleinmann, *Infinite Polyhedra,* Technion, Haifa, 1974.
[2] Rinus Roelofs, *The discovery of a new series of Uniform Polyhedra*, Bridges Proceedings, Enschede, 2013.