About Weaving and Helical Holes

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Abstract

Weaving is an invention of man, and maybe one of the most important inventions in the field of construction. There are many weave patterns but most weave patterns are based on just two different grids, the plain weave and the three way weave. The development of weaving is mainly connected to the possibilities of manufacturing. In this paper I want to investigate this 'passing one thread over another'-concept from a mathematical-artistic point of view.

1. Introduction

1.1. Over and Under. In Leonardo's painting on the ceiling of the Sala delle Asse in the Sforza Castle in Milan (Figure 1) we can see different weaving patterns: the knotted ropes and the interwoven branches. Both can only be realized with the aid of man. Nature doesn't weave by itself, at least not in the way Leonardo shows us in his painting. When we look for a definition of weaving we can find the following: weaving is the textile art in which two distinct sets of yarns or threads, called the warp and the filling or weft, are interlaced with each other to form a fabric or cloth. To define the concept of weaving in a more general way we could say that a weaving is a line pattern in which for each pair of adjacent crossing points on the line(s) the position of that line changes from 'over' to 'under' or from 'under' to 'over' (Figure 2a,b).

Figure 1: *Leonardo da Vinci*

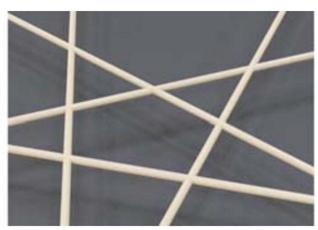


Figure 2a: *Lines*

Figure 2b: *Weaving*

1.2. Graphs and Tilings. Lines in a weave pattern can be lines with a start and an endpoint (Figure 2b) or closed loops (Figure 3). And a weave pattern can even be made with one single line (Figure 4a). When we project a weave pattern on the plane the resulting figure can be seen as a graph in which each vertex represents a crossing point of the weave pattern.
The degree of each vertex is four, because of the two lines in the weaving pattern crossing each other. And therefore the projected pattern can be transformed into a tiling which can be colored with just two colors in such a way that adjacent tiles always have different colors (Figure 4). This works both ways, so we can construct a weave pattern from any tiling that can be colored in this way and has only vertices with degree four.

Figure 3: *Closed loops.*

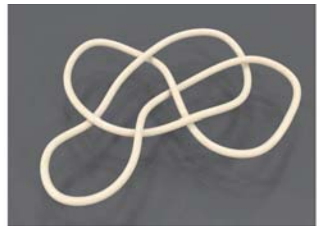


Figure 4a: *Weaving.*

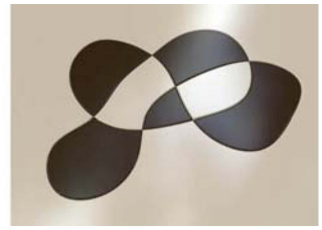
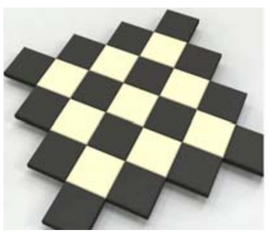
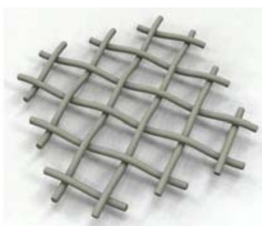


Figure 4b: *Tiling.*

2. Weave Patterns

2.1. Tilings and Weave Patterns. When we look at the Archimedean tilings [1] we see that there are four tilings which can be colored in the way described above. Two of them, (4.4.4.4) and (3.6.3.6) are the underlying patterns of most of the weavings used in fabrics and baskets. The weaving derived from (4.4.4.4) is known as the plain weave (Figure 5), and the other as the three way weave (Figure 6).





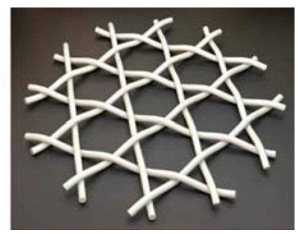


Figure 5a: *Tiling 4.4.4.4.* **Figure 5b**: *Plain weave.* **Figure 6a**: *Tiling 3.6.3.6.* **Figure 6b**: *Three way weave.*

2.2. Closed Loops. The third Archimedean tiling that can be colored like a checkerboard is (3.4.6.4) (Figure 7a). The weaving derived from this tiling is that it is a weaving with closed loops (Figure 7b).

Figure 7a: *Tiling 3.4.6.4* **Figure 7b**: *Weaving with closed loops.*

The last one, tiling (3.3.3.3.3.3), can be colored in the right way but can not be transformed into a weaving pattern because the degree of each vertex is not four but six.

3. Transforming Patterns into Two-Color Patterns

3.1. Connecting intersection points. Each tiling can be used to create a new tiling that can be colored as a checkerboard. When we add the dual pattern to an existing pattern we will get intersection points between both patterns. Each of those points lays on a line which is the edge of two adjacent tiles. On both tiles this

edge is connected to exactly two other edges, on which there are also intersection points. So we have four direct neighbour points for each intersection point. Connecting each intersection point to the four direct neighbour points will thus give us a graph in which each vertex has degree four. So we will get a tiling which can be colored as a checkerboard, that can be used as an underlying pattern for a weaving. In Figure 9 you can see the result of this process applied on the Archimedean tiling (3.3.4.3.4) (Figure 8).
This operation corresponds to the "ambo" operation that is part of Conway Polyhedron notation [2].

Figure 8a: *Tiling.*

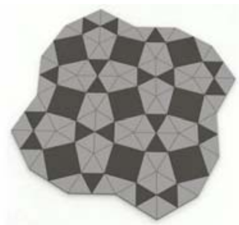


Figure 8b: *Transformed tiling.*

Figure 9: *Weave.*

3.2. The other Archimedean Tilings. As can be seen in Figure 10 to 13 we can derive new weave patterns from the other Archimedean tilings. Note that when we apply this process on tiling (3.3.3.3.3.3) or on tiling (6.6.6) in both cases we will get tiling (3.6.3.6), which already explained in Section 2.1. Starting with the tiling (4.4.4.4) brings us back to tiling (4.4.4.4) again.

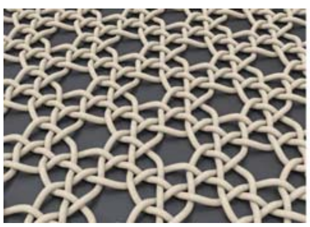


Figure 10: *(6.3.3.3.3).*

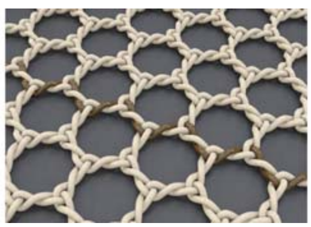


Figure 11: *(3.12.12).*

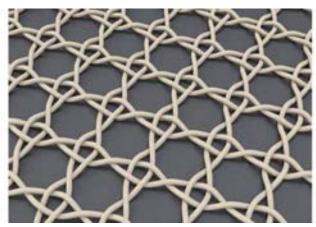


Figure 12: *(4.8.8).*

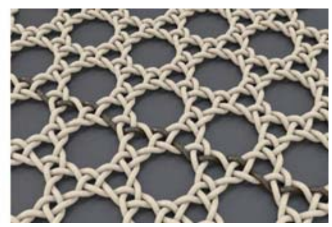


Figure 13: *(4.6.12).*

4. Weaving Patterns on Polyhedra

4.1. Rings on the Sphere. In the group of regular and semi regular polyhedra there are, except from the anti prisms, five polyhedra that can be colored in a checkerboard fashion. And they all have vertices with degree 4. This gives us five weave patterns on the sphere as can be seen in Figure 14.
In Japanese Temari Balls and also in Alan Holden's Orderly Tangles [3] we can find many examples of the use of these basic patterns.
But with the procedure described in section 3.1 we can also derive weave patterns from the other Platonic and Archimedean solids. In Figures 15 to 20 some examples are shown. In the weavings we still get closed loops but these lines are not laying in a cutting plane of the polyhedron as in the first five. And in some cases (snub cube and snub dodecahedron), the loops go around twice, crossing themselves a few times.
The weavings shown in Figures 15 – 20 are based on the Rhombic Truncated Cuboctahedron, the Truncated Dodecahedron, the Rhombic Truncated Icosidodecahedron, the Snubcube and the Snubdodecahedron.

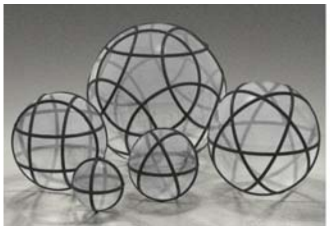


Figure 14: *Five spherical weaves.*

Figure 15: *R.T.Cuboc.*

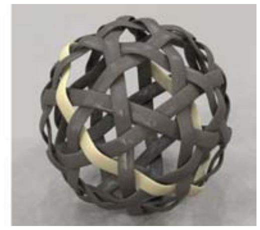


Figure 16: *T.Icos.*

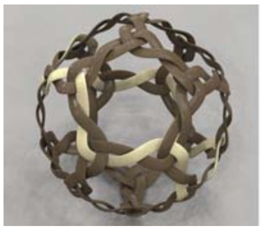


Figure 17: *T.Dodec..*

Figure 18: *R.T.Icosidodec.*

Figure 19: *Snub Cube*

Figure 20: *Snub Dodec.*

5. Twist

5.1. Transformations. As tilings can be used to generate weaving patterns, also weaving patterns themselves can be used to generate other weaving patterns. We will discuss and illustrate some of the possible transformations that can be used to generate new weaving patterns and also new weaving structures.

5.2. From lines to closed loops. In a plain weave there are straight crossings between the lines. You can say that each line at each crossing point takes the second exit. If we change that, so instead of the taking the second exit we take the third and we do that with both lines of the crossing, we will get a twist at the crossing point of our weaving. The first twist (Figure 21a) will just cause a defect in the weaving. But applying this twist on every crossing point in the weaving will transform our line weaving into a weaving with closed loops (Figure 21c). Figure 22 shows the result of this transformation starting with the three way weave.

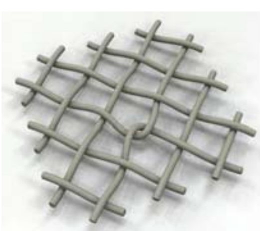


Figure 21a: *One defect*

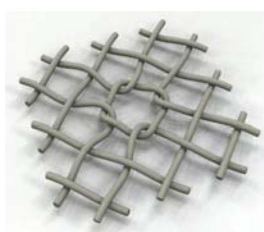


Figure 21b: *One loop.*

Figure 21c: *Loop* weaving.

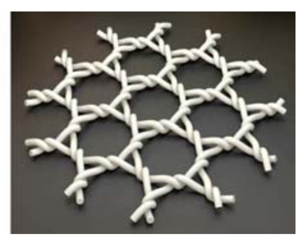


Figure 22: *Three way.*

5.3. Change of Rhythm. The next operation can be seen as a change of rhythm: instead of over-under-over-under (notation: 1-1) we will change to over-over-under-under (notation: 2-2). To achieve this we just change some of the crossings from over-under to under-over. In Figure 23 you can see the result of this operation applied on the weaving of Figure 21c.

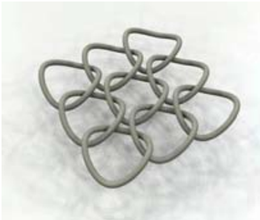


Figure 23: *Rhythm 2-2.*

Figure 24a: *Rhythm 3-3.*

Figure 24b: *Rhythm 4-4.*

Figure 24c: *Surface.*

The rhythm of the weaving with the ellipses in Figure 24a is 3-3 and when we make an extra twist in each of the ellipses we will get the weaving of Figure 24b in which the rhythm is 4-4.

5.4. Making Surfaces. It appeared to be possible to create a surface on the edges of the weaving of Figure 24b like a soap-film. Whether this is due to the fact that the rhythm is even-even (4-4) or if this is possible with all even-even loop weavings still has to be examined. The resulting surface however is a self interwoven surface and as such a new weaving structure derived from a weaving pattern. In Figures 25 to 27 a few more examples are shown of self interwoven surfaces created from twisted loop weavings. The starting point for Figure 25 is the three way weave. In Figure 26 the Archimedean tiling (3.4.6.4) can be recognized. And to construct the surface of Figure 27 one more step was needed: starting with the Archimedean tiling (3.4.3.3.4) we first had to use the method described in section 3.1. In the surfaces the lines of the twisted ellipses can now be seen as 'twisted holes'. In my paper "Connected Holes" [4] similar self interwoven can be found. The difference is that shape the holes now is just a twisted closed loop and not a knot, which was used as the shape of the holes in the surfaces explained in "Connected Holes".

Figure 25a: *Weave.*

Figure 25b: *First twist.*

Figure 25c: *Next twists.*

Figure 25d: *Twist weave.*

Figure 25e: *Surface.*

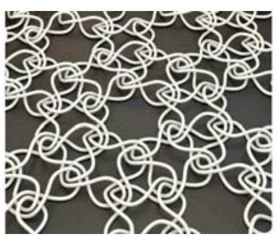


Figure 26a: *Weave.*

Figure 26b: *Surface.*

Figure 27: *Surface.*

6. Weaving with Surfaces

6.1. Soap Film. In Figure 23 we have shown a loop weave with rhythm 2-2. This is an even-even rhythm and also here it appeared to be possible to create surfaces on the edges using the soap-film method. However in this case the complete set of loops of the weaving splits up in two subsets, creating two surfaces which are interwoven, as can be seen in Figure 28.

Figure 28a: *Subset.*

Figure 28b: *Soap film.*

Figure 28c: *Surface.*

Figure 28d: *Weaving.*

6.2. Examples. This is in fact a new class of weaving structures. Two or more surfaces can be interwoven when there are holes in the surfaces. The complete set of holes corresponds to a loop weave pattern. Many different weavings of this kind can be made but for now I just want to show some examples.

Figure 29: *Square grid.*

Figure 30: *Square grid.*

Figure 31: *Hexagonal grid.*

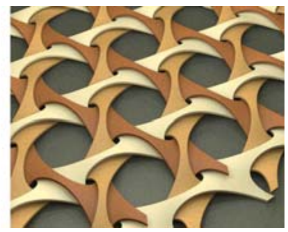


Figure 32: *Three layers.*

Figure 33: *Three layers*

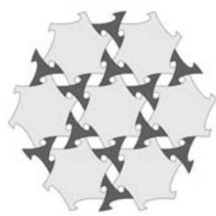


Figure 34: *Two layers.*

Figure 35: *Two layers.*

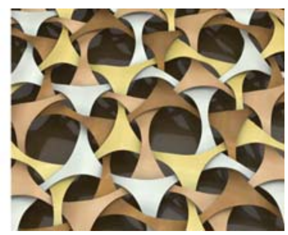


Figure 36: *Four layers.*

7. Helical Holes

7.1. Twist again. So far we have seen that 'twist' is the base of some powerful operations to generate new weaving patterns as well as new weaving structures. Until here all the operations were applied on line or loop weaves. But we can also define a twist operation that can be applied on weave surfaces. In Figure 37a you can see a part of the weave surface that is created in Figure 28. It is a part that surrounds one hole in the surface. Or the part that is connected to one loop in the original weave. When we split up this part into four pieces (this is the number of surrounding holes) we can twist one part over 360/4 degrees. In Figure 37b you can see that the edge of the hole is now transformed into a non-closed line and after twisting one more part this line is transformed into a helix (Figure 37c). The unit we now have created can be used to create a surface as shown in Figure 37d. In fact such a surface can be extended into infinity in all directions. And special about this surface is that it has helical holes.




Figure 37a: *One hole.* **Figure 37b**: *First twist.* **Figure 37c**: *Helix.* **Figure 37d**:
Surface.

7.2. Weaving with Helical Hole Surfaces. Another property of these surfaces is that they can be used to make weaves again. We can weave two or more of these surfaces through one another. And there are also several ways to do this. In Figure 38b an extra surface is added which is moved horizontally to create a weave that can be compared with the weave in Figure 28d. The other way of weaving is adding a copy which is translated in the vertical direction (Figure 39). Both operations can be combined to create a four-layer weave with helical hole surfaces. As you can see in Figure 39d there is still space to add more layers in between the two layers. The number of possible layers depends on the thickness of the surface and the pitch of the helix.

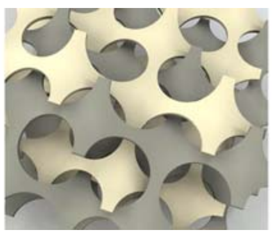



Figure 38a: *S urface* **Figure 38b**: *Weaving a.* **Figure 39a**: *Surface.* **Figure 39d**:
Weaving b.

7.3. Examples. Any tiling that can be coloured as a checkerboard can be transformed into a weaving pattern, but only when the degree of all the vertices is four. It turns out that all such tilings also can be transformed into spiral hole surfaces, but now also for degrees of the vertices higher then four. In Figure 40 we see a helical hole surface based on the Archimedean tiling (3.3.3.3.3.3), a tiling with vertex degree 6. Figure 40c and 40d respectively show the weaving of two and three surfaces. The helical hole surface of Figure 41 is based on the Archimedean tiling (3.4.3.6).



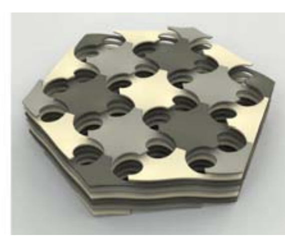



Figure 40a: *Surface.* **Figure 40b**: *1 Layer.* **Figure 40c**: *2 Layers.* **Figure 40d**: *3*
Layers.

Figure 41a: *Surface.*

Figure 41b: *1 Layer.*

Figure 41c: *2 Layers.*

Figure 41d: *3 Layers.*

8. Weave Grids 3D

8.1. Black/White – Left /Right. In his article "Weaving, Mother of Tensegrity" [5] Kenneth Snelson writes, and he illustrates it with the picture that is copied here as Fig 42b: "Each woven interaction produces its rotational complement. Just as the individual crossings of filaments have their helical axes so each square in a plain weave has its opposite. Each cell's neighbours are its mirror form like alternate squares on a chess board." And this is exactly what is happening in the helical hole surfaces: neighbouring holes are each other's mirror images. If one spiral hole is clockwise, his neighbour hole is counter clockwise. When we make this relation visible like in Figure 42c and 42d we can apply the soap film method again to create the surface shown in Figure 42e.

Figure 42a: *Tiling.*

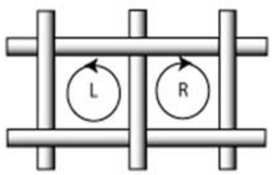


Figure 42b: *Snelson.*

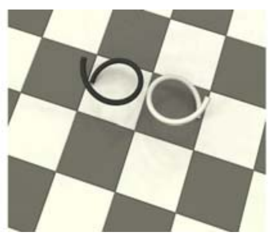


Figure 42c: *Left-Right.*

Figure 42d: *Spirals.*

8.2. Black/White Tiling on Helical Hole Surfaces. One of the important discoveries Kenneth Snelson talks about in his article is 3D weaving. The examples he shows are based on space frames. Also helical hole surfaces can be used to generate 3D weaves. And this method is in a way exactly the same as the generation of the weaves from the checkerboard colored tilings that we have discussed in Sections 2 and 3: as you can see in Figure 42f the helical hole surface can be covered with a black/white tiling by projecting the underlying coloring on it. And this tiling can be transformed into a weaving again. It all looks the same but there is one big difference: the weaving that we have created now is a 3D weaving. Each thread in the weaving is connected with threads at two different levels.

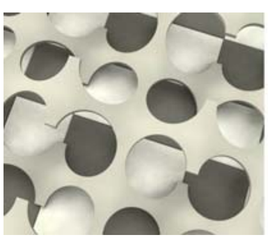


Figure 42e: *Surface.*

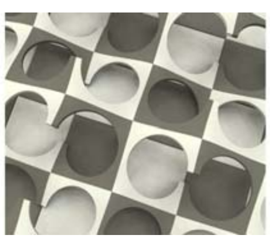


Figure 42f: *Projecting.*

Figure 42g: *Tiling.*

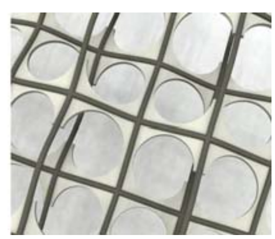
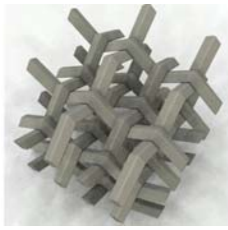


Figure 42h: *3D Weaving.*

8.3. 3D Weaves. To make this more visible I have made two examples of these 3D weaves. The first one is based on the Archimedean tiling (4.4.4.4) and the second one is based on the tiling (3.6.3.6).



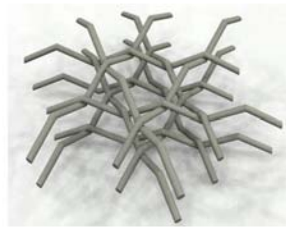


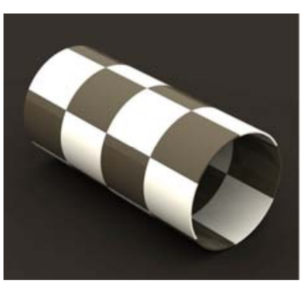
Figure 43: *3D weave square.* **Figure 44**: *3D weave hexagonal*

9. Helical Hole Surfaces.

9.1. Black/White Tilings. In Figures 42a – 42e we have shown that helical hole surfaces can be created starting with a checkerboard tiling. And in Chapter 4 we have seen that also some of the regular and semi regular polyhedra can be colored in a checkerboard fashion. So when we use the same steps as in section 8.1, but now starting with the icosidodecahedron (Figure 45), we can create the spherical helical hole surface shown in Figure 46. Note that this is a single surface structure. In Figure 47 we can see another example of a spherical helical hole surface. This one is based on the cuboctahedron.

Figure 45: *Icosidodecahedron* **Figure 46**: *Helical Hole Surface* **Figure 47**: *Base: Cuboctahedron.*

9.2. Cilindrical Tilings. Also cylinders can be covered with a black/white tiling. In Figure 48 and Figure 50 we can see two basically different coverings of the cylinder. In Figure 50 the covering is made in a spiral way. Both coverings can be used to create a helical hole surface (Figure 49 and 51).



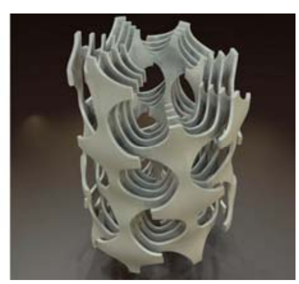


Figure 48: *Cylinder* **Figure 49**: *Helical Hole Surface* **Figure 50**: *Spiral covering* **Figure 51**: *Surface*

Both the spherical and the cylindrical helical hole surface can be interwoven with copies of itself (see Section 7.2). Those copies however have to be a scaled version of the original surface. We can solve this by starting with a cone instead of a cylinder. In that case the scaling is introduced in the black/white tiling. And therefore we can create a helical hole surface of which a copy will fit in the empty space of the original surface.

9.3. Fibonacci Tilings. There is also a way to introduce the scaling of the tiles in a flat tiling. For that we have to look at patterns that are based on the Fibonacci numbers. When we use the logarithmic spiral which is related to the Fibonacci numbers, we can create tilings in which each tile is a scaled copy of any of the other tiles. All tiles have the same shape but not the same size. These so called Fibonacci tilings can also be found in nature. A well known example is the pattern of the seeds in a sun flower. The net of such a tiling is the intersection of two sets of logarithmic spirals. If we choose the right combination of the number of spirals in each of those sets we can get a tiling that can be colored in a checkerboard fashion. In the example in Figure 52 there are 7 spirals in one set and 9 spirals the other. Based on this tiling we can construct an helical hole surface.

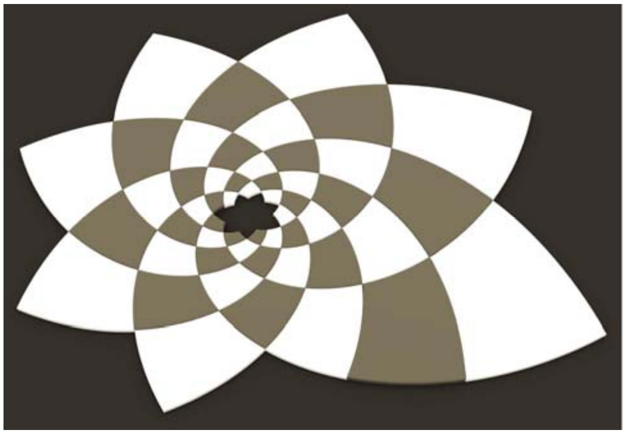


Figure 52: *Fibonacci Tiling*

Figure 53: *Spiral Helical Hole Surface*

And now we have got a spiral helical hole surface (Figure 53) that can be interwoven with an exact copy of itself.

10. Conclusion

10.1. Weaving and Helical Hole Surfaces. In architecture most of the ideas for constructions are based on nature. Weaving is invention of mankind and a lot of possibilities are still to be discovered. The discovery of the spiral hole surfaces leads to organic looking structures based on patterns that we know from nature. In a way it brings us back to the question: Does nature weave? The 3D weave structures of Kenneth Snelson have inspired me very much. This is my attempt to explore this a little further. I realize it is just a start.

References

[1] Grünbaum and Shephard, *Tilings and Patterns,* W.H. Freeman and Company, New York, 1987.
[2] http://en.wikipedia.org/wiki/Conway_polyhedron_notation
[3] Alan Holden, *Orderly Tangles,* Columbia University Press, New York, 1983.
[4] Rinus Roelofs, *Connected Holes,* in Bridges Leeuwarden Proceedings, 2008
[5] Kenneth Snelson, *Weaving, Mother of Tensegrity,* www.kennethsnelson.net