A Mistake in a drawing by Leonardo da Vinci

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1. Divina Proportione

Luca Pacioli. In 1509 Luca Pacioli's book Divina Proportione (ref. 1) was published. A big part of the book is dedicated to the regular and semi-regular polyhedra, the Platonian and Archimedean solids. The illustrations for these figures were made by his friend Leonardo da Vinci.

Figure 1: *Luco Pacioli and his book Divina Proportione.*

Besides the thirty-eight drawings on the regular and semi regular polyhedral there are about twenty drawings on spheres (Fig. 1), columns and pyramids.

2. The Platonic Solids

Da Vinci's drawings can be arranged in a few ways. They can be divided in two groups, the Platonic (Fig. 2) and the Archimedean solids (Fig. 3).

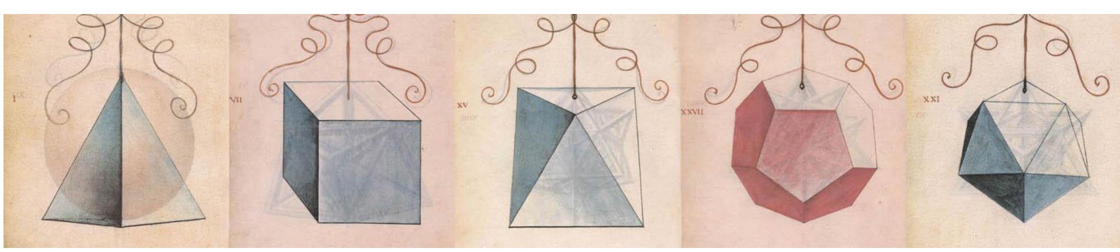

Figure 2: *The Platonic Solids.*

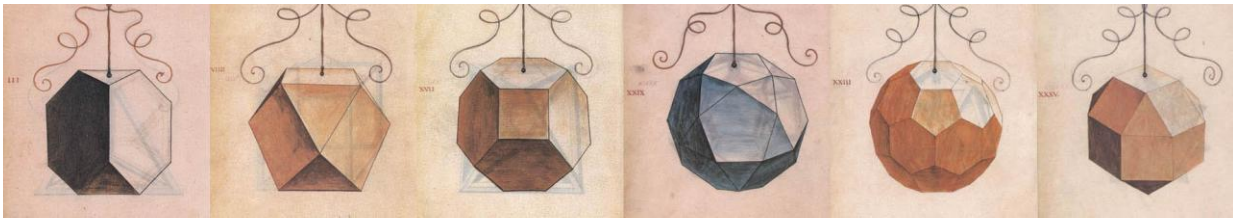

Figure 3: *The Archimedean solids in Divina Proportione.*

For all solids Leonardo used more than one ways to illustrate the construction of the solid, as you can see in the example of the dodecahedron in Figure 4. We will come back to this later.

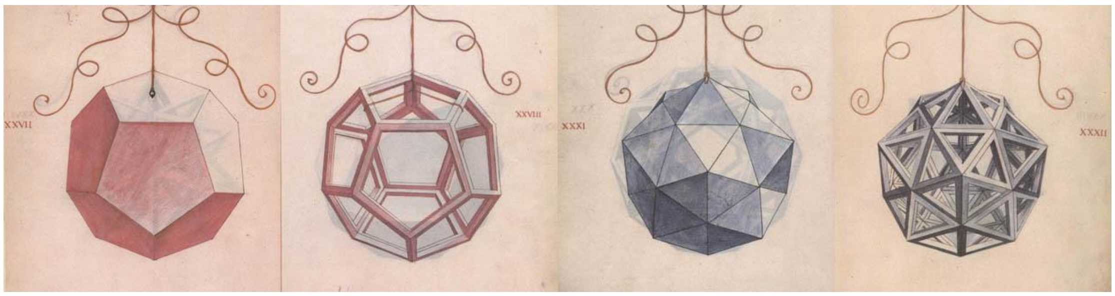

Figure 4: *Four versions of the Dodecahedron: Solidus, Vacuus, Elevatus Solidus and Elevatus Vacuus.*

3. The rediscovery of the Archimedean Solids in the Renaissance

Luca Pacioli only describes six of the thirteen Archimedean solids. This is due to the fact that in the Renaissance the Archimedean solids have been rediscovered. The story of the rediscovery is very well described by J.V. Field (ref. 2).

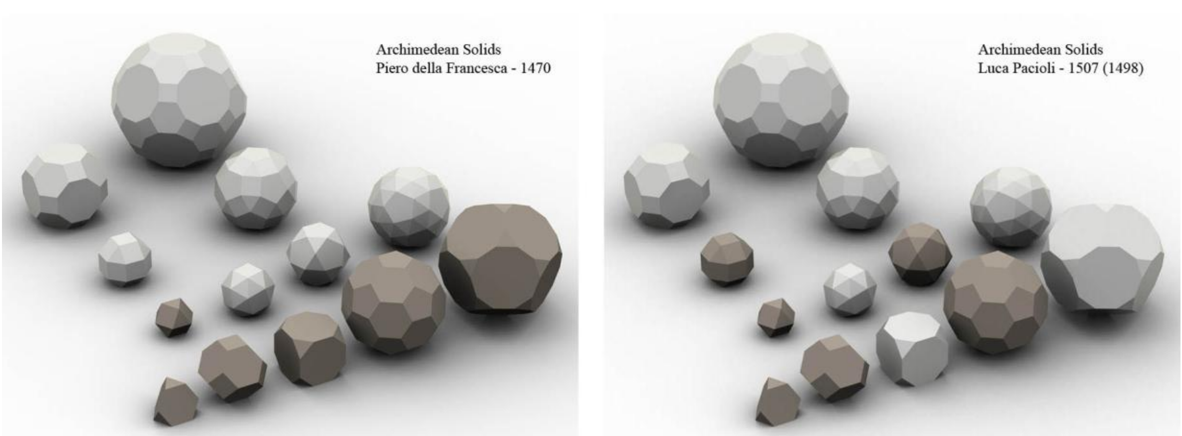

Figure 5: *Archimedean solids – Piero della Francesca – Luca Pacioli*

The first publication in the Renaissance about polyhedra was done by Piero della Francesca. He described six of the thirteen Archimedean solids, and these are all truncations of the Platonic solids. In the book of Luca Pacioli, who was a student of Piero della Francesca, we also see six Archimedean solids, but there are two new ones and two others were left out. You can find a solid which is called truncated cube but this is in fact the cuboctahedron. And so is, in Luca Pacioli's book, what he calls the truncated dodecahedron, the solid we know as the icosidodecahedron. The other new solid in his collection is the rhombic cuboctahedron.

4. Elevation

A new and interesting concept in Pacioli's book is a transformation which he calls Elevation. It can be described as follows: take the centre-point of each face and raise it till you can make a pyramid with equilateral triangles connecting the raised point with each of the edges of the original face (Fig.6). It is easy to understand that this can only be done with triangular, square and pentagonal faces.

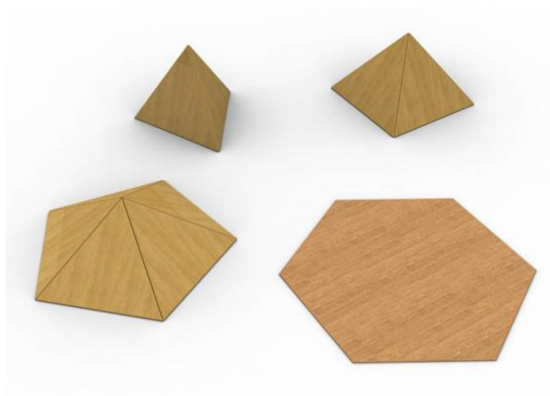

Figure 6: *Elevation of the triangle, square and pentagon.*

So for three (Fig. 7) Archimedean solids this transformation cannot be applied because of the hexagonal faces.

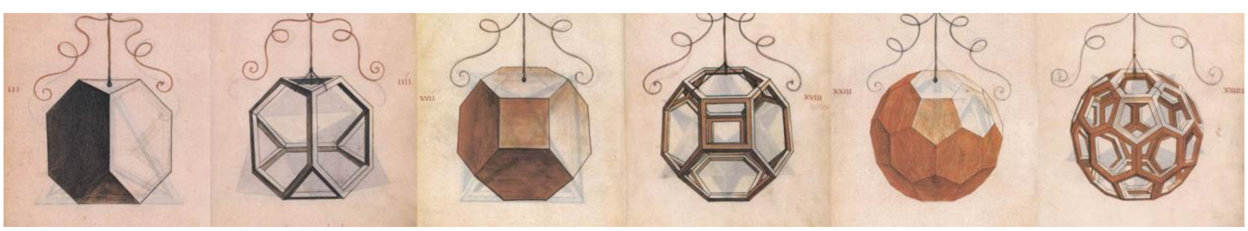

Figure 7: *Archimedean solids with hexagonal faces.*

You can see the illustrations, including the elevated versions, of the other three, the cuboctahedron (In Paciloi's book referred to as Exacedron Abscisus (truncated cube)), the icosdodecaheron (Duodecedron Abscisus) and the rhombic cuboctahedron (Vigintisex Basium) in Figures 8, 9 and 10.

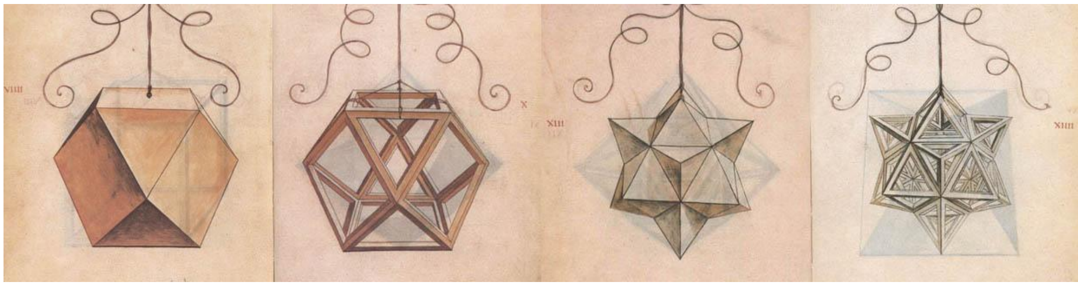

Figure 8: *Four versions of the Cuboctahedron.*

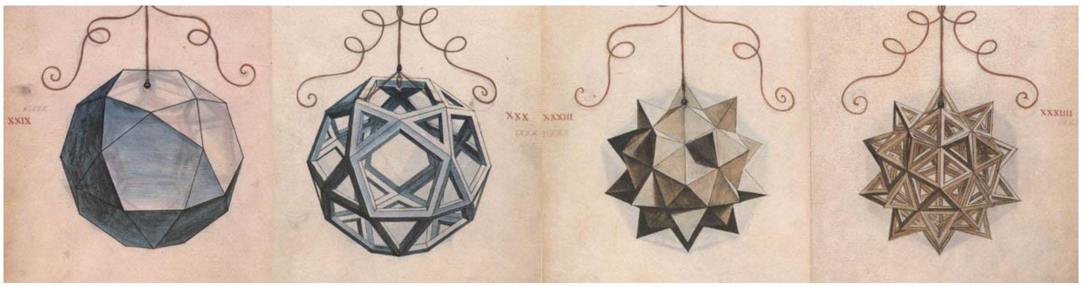

Figure 9: *Four versions of the Icosidodecahedron.*

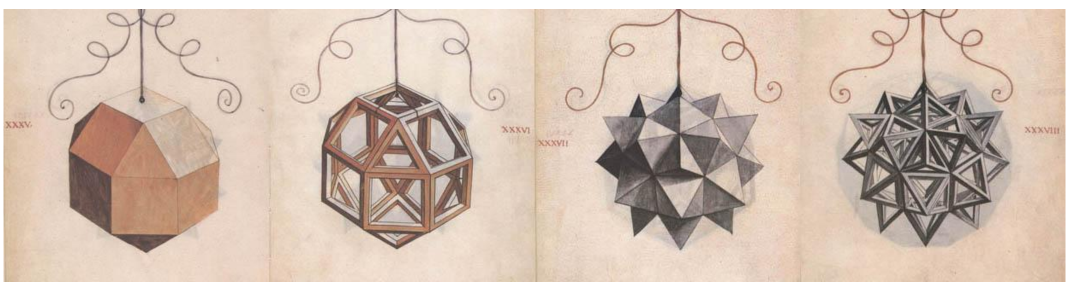

Figure 10: *Four versions of the Rhombic Cuboctahedron.*

5. The Mistake by Leonardo da Vinci

The drawings of the elevated versions of the rhombic cuboctahedron (Vigintisex Basium, 26 faces) (Fig. 10) are without any doubt a few of the most complex drawings of the book. As can be seen on the signs that Leonardo has drawn above each of the drawings (Fig. 11) it is sure that this is a serie based on the mathematical structure of the Archimedean solid, the rhombic cuboctahedron.

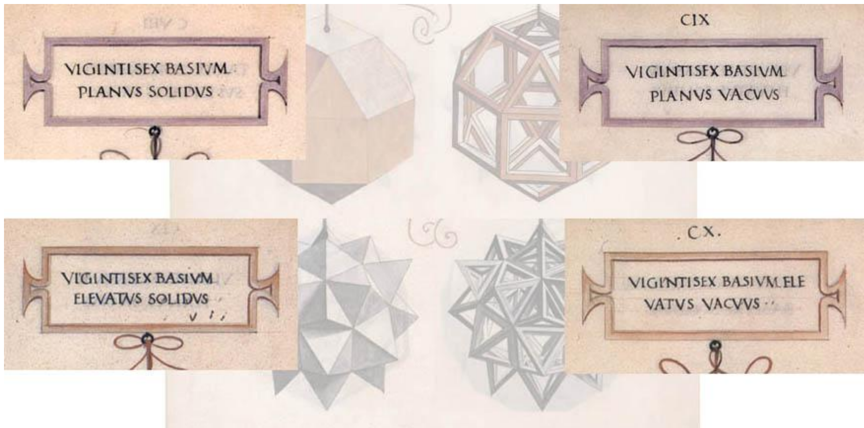

Figure 11: *Labelling of the four versions of the Rhombic Cuboctahedron.*

Because of the complexity I have made a computer-drawing of Leonardo’s version of the elevated rhombicuboctahedron. And after comparing the two drawings it is clear that there is a mistake in Leonardo’s drawing.

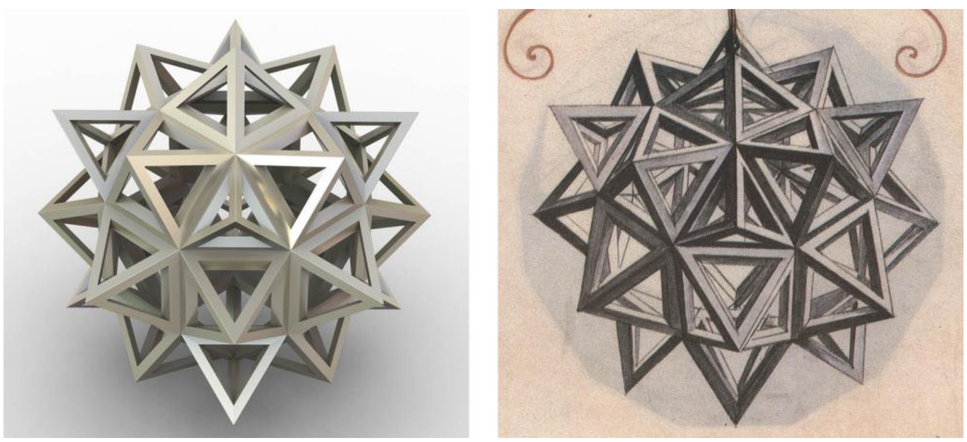

Figure 12: *Computer drawing of the RCO and Leonardo’s original drawing of the RCO.*

Zooming in at the bottom part of the drawing shows us that in Leonardo’s drawing a four-sided pyramid is placed in the position where we had a triangle face in the non-elevated version.

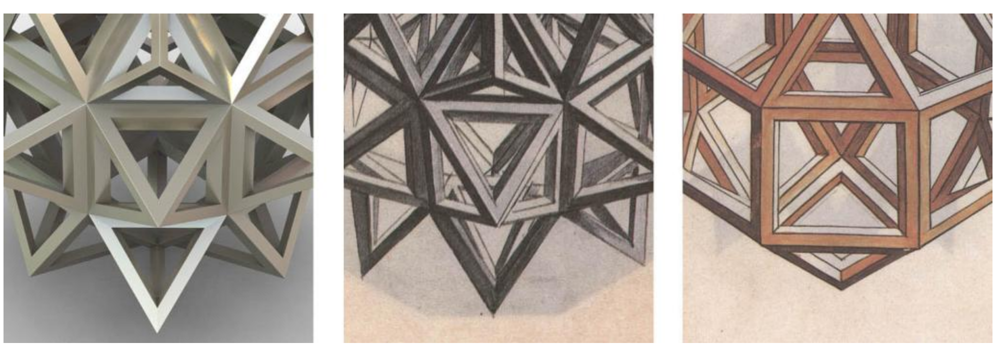

Figure 13: *The mistake in Leonardo’s drawing.*

And, maybe as a consequence of the mistake, we see triangular pyramids where we would expect four-sided pyramids at the left-bottom and right-bottom parts of the drawing.

6. Further Rediscovery of the Archimedean Solids.

A few years after Pacioli's book Albrecht Durer published his book about geometry (first published in 1525). He describes a few new "rediscovered" polyhedra, among which the snub-cube. He did not make perspective drawings but he shows us the nets of the polyhedral, as you can see in the example of Figure 14.

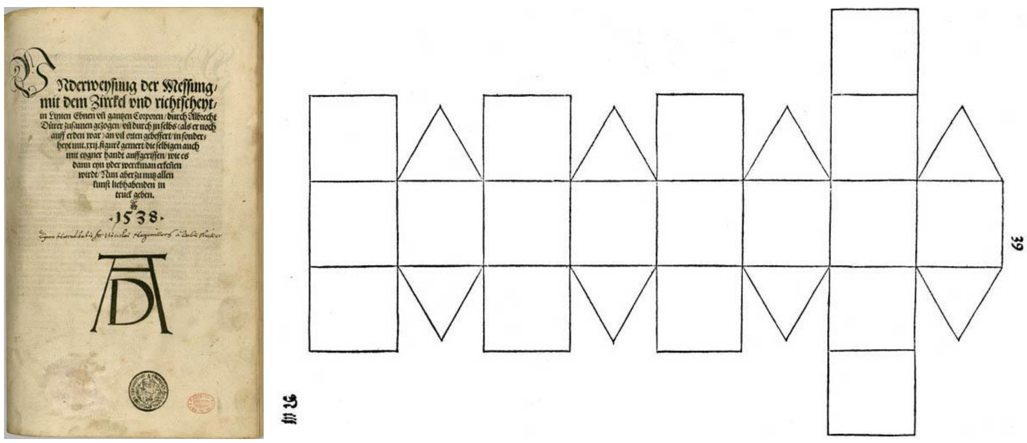

Figure 14: *Durer's book and his drawing of the layout of the RCO.*

Figure 15: *Further rediscovery of the Archimedean solids – Durer – Barbaro.*

With Barbaro's publication in 1568 the list was almost complete (Fig. 15), but it was Kepler who in 1619 published all thirteen Archimedean solids (Fig. 16) in his Harmonices Mundi.

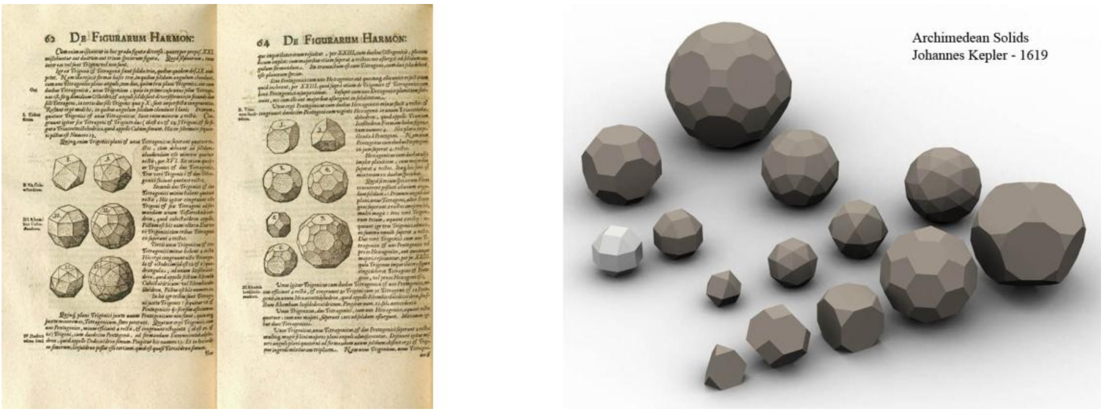

Figure 16: *Kepler*

Till the beginning of the twentieth century it was always believed to be true that there were 13 Archimedean solids, convex solids with only regular faces of more than one kind and for which each corner point is surrounded by the same configuration of faces. As far as we know there is no classical text by Archimedes but Pappus stated (in his "Collection" – AD 300-350) that Archimedes had found the thirteen solids, and he gave a description of all of them. And then J.P. Miller came up with a new solid that completely satisfies all the criteria mentioned above. As Coxeter writes in book "Mathematical Recreations and Essays" the new Archimedean solid was discovered by mistake.

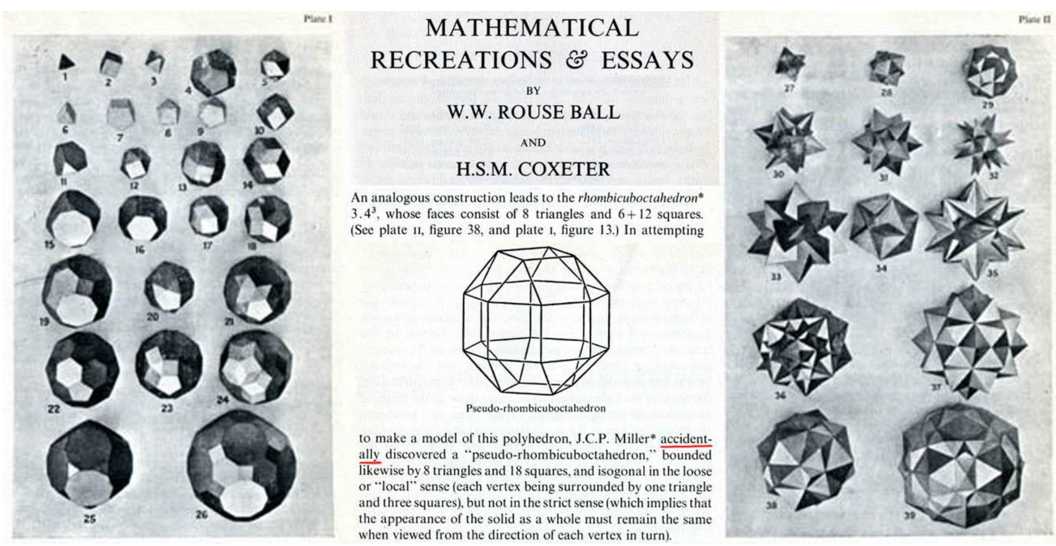

Figure 17: *Coxeter about the discovery of the pseudo-rhombicuboctahedron.*

7. Possible Explanation of Leonardo's Mistake

Because this new discovered polyhedron can be seen as the Rhombic Cuboctahedron of which the lower part has been rotated, the position of the squares and the triangles in the lower part is exchanged. When we look back at Leonardo's drawing now, is it possible that he made a mistake like this, and in fact made a drawing of the pseudo rhombic cuboctahedron instead of the normal rhombic cuboctahedron? Of course this assumption still means that he made a mistake because of the sign he had drawn in the upper part of the illustration, but we have to examine this possibility. The difference between the two solids becomes very clear when we first start with the elevation of the triangular faces (Fig. 19).

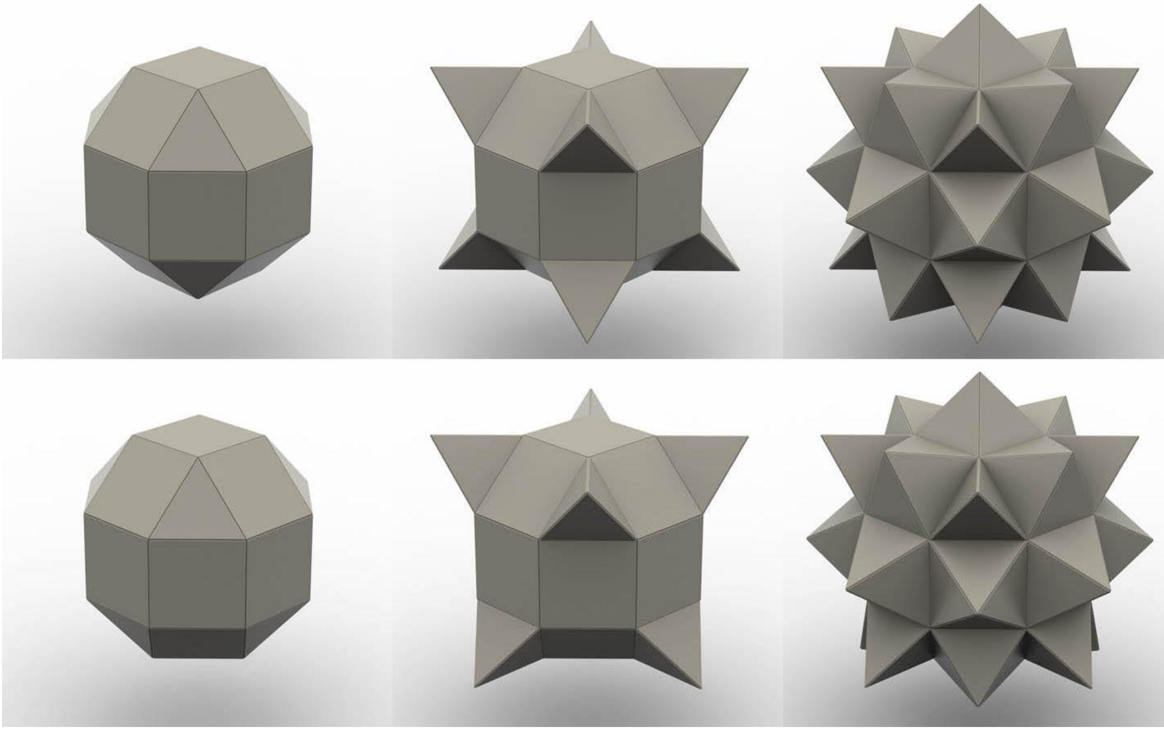

Figure 19: *Elevation of the pseudo rhombic cuboctahedron*

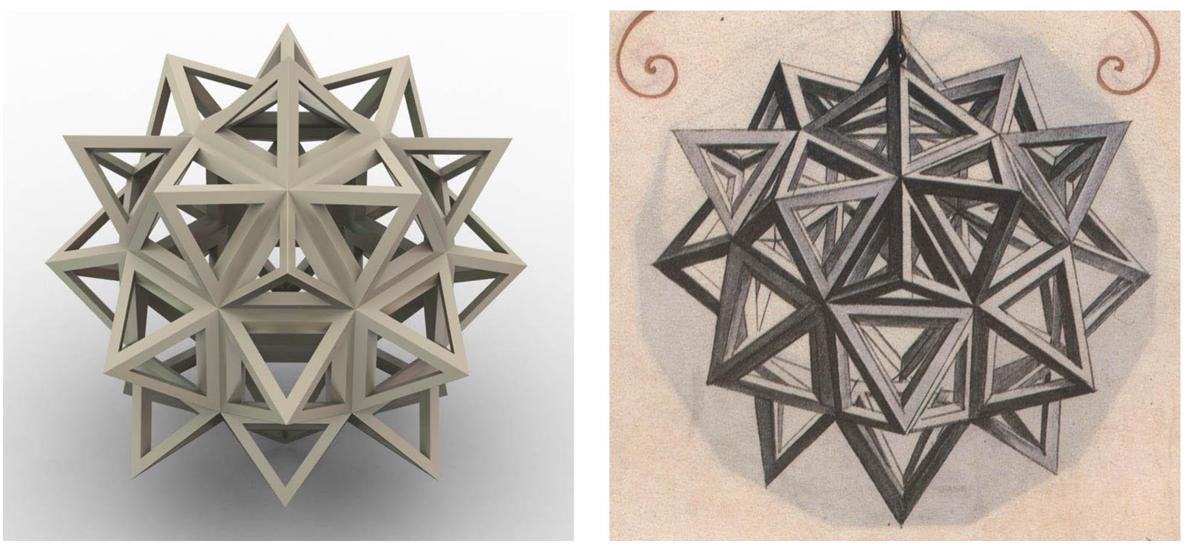

Figure 20: *Is the solid on Leonardo's drawing the pseudo rhombic cuboctahedron?*

In Figure 20 we can compare the computer drawing of the pseudo rhombic cuboctahedron with Leonardo’s drawing of the rhombic cuboctahedron. Although at first glance both pictures may look similar we still see big differences. When we take away some parts from the computer-image (Fig. 21 right) we see that the most lower pyramid, the pyramid that stands on the ground is either completely missing in Leonardo’s drawing or is the most lower square pyramid in his drawing. But then the triangular pyramid is missing.

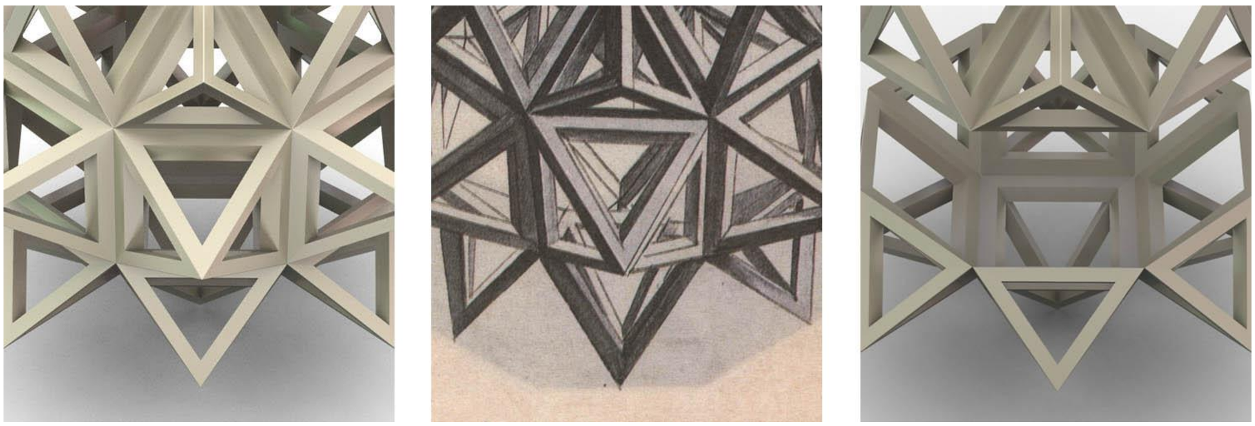

Figure 21: *Leonardo’s drawing compared with the computer-drawing of the pseudo RCO.*

8. Physical Models of Leonardo’s Drawings

The polyhedral drawings made by Leonardo appeared to be very inspiring for model makers. Many examples of real build models after the drawings are known. The most impressive collection is made by the Italian artist Adriano Graziotti. Between 1960 and 1980 he constructed many polyhedral models, not only the ones described by Luca Pacioli, but also the later rediscovered Archimedean solids. As you can see in the pictures it is a huge collection and it can still be seen in Rome in the museum of the University.

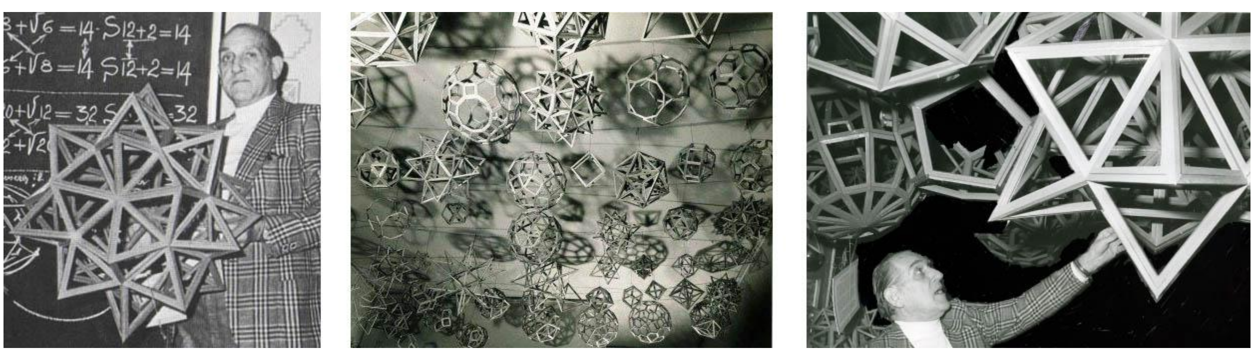

Figure 22: *Graziotti and his models*

A remarkable example is the model of the elevated rhombic icosidodecahedron which he did in the Leonardo style.

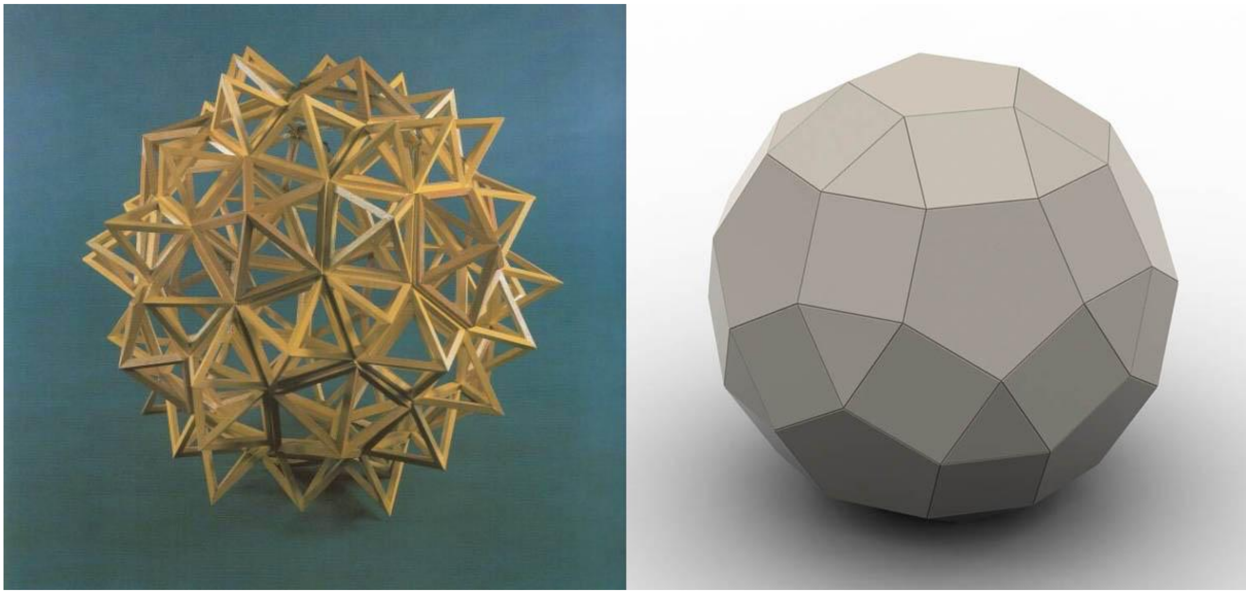

Figure 23: *Elevated rhombic icosidodecahedron*

Just like in the rhombic cuboctahedron a section of the solid can be rotated to create a new solid. However in this case we do not get a new Archimedean polyhedron, because each corner point is not surrounded by the same configuration of faces. At some points it is now square, square triangle, pentagon instead of square, triangle, square, pentagon (Fig. 24 right).

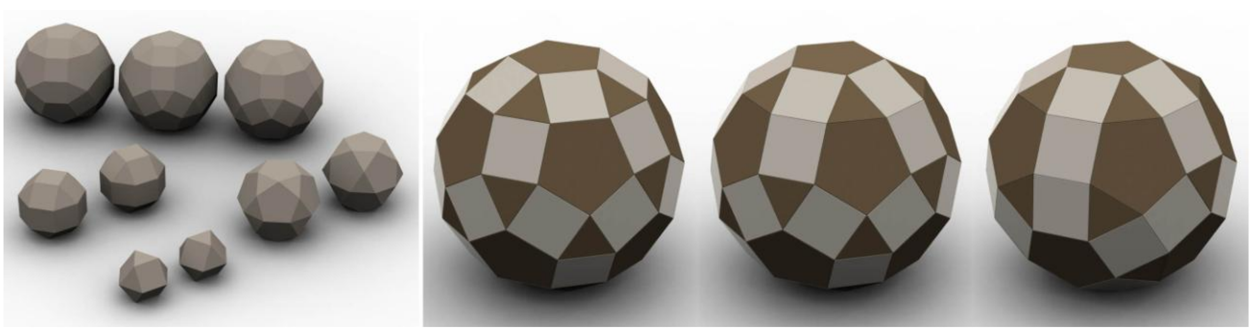

Figure 24: *From some of the Archimedean solids we can create new solids by rotating a section.*

Figure 25 (left) shows us the drawing that Graziotti made before he constructed the real model. In the text above the drawing we can read: “In stellating this polyhedron we followed the method discovered by Leonardo da Vinci.” Looking close at his drawing we see that also Graziotti made a mistake here. In the mid-part of the upper section we see two connected square pyramids which means that he has used one of the rotated versions of the rhombic icosidodecahedron.
In the real wooden model this mistake seems to be corrected (Fig. 25 right).

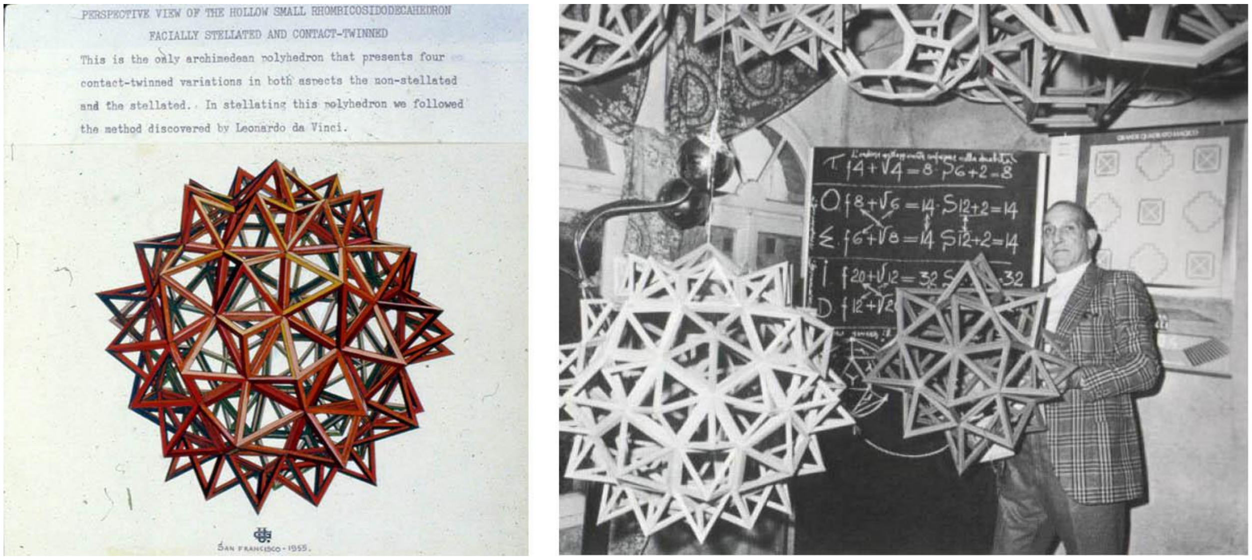

Figure 25: *Graziotti and his model of the rhombic icosidodecahedron.*

9. Errors in the Da Vinci Museum

Making mistakes is part of the creative process. Also in science we see that mistakes quit often turn out to be a starting point of a new discovery. In Leonardo's case, maybe he could have discovered the fourteenth Archimedean solid before Miller did it in 1907. But in some cases mistakes have to be avoided. In the Leonardo da Vinci Museum in Vinci you will find one room dedicated to the polyhedral models. Wooden models in the style of Leonardo are hanging on the ceiling and the walls are covered with plates to explain how to build polyhedral models yourself.

Figure 26: *The polyhedron room of the Leonardo da Vinci museum.*

One plate shows us how to cut out a plan of eight triangles for the construction of the octahedron. There are eleven ways to draw a configuration of eight triangles in such a way that you can fold an octahedron out of it. The one on the plate of the museum is different from all eleven. Did they find a twelfth way or is it a mistake?

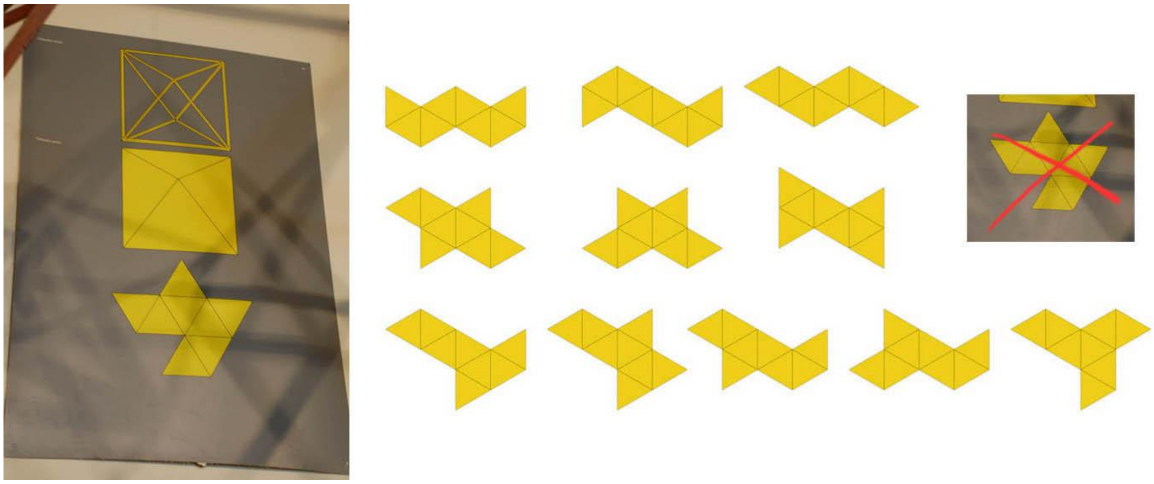

Figure 27: *Plans of the octahedron.*

Trying to fold a three dimensional object from the plan given by the museum results in boat-like shape instead of the octahedron. An error that shouldn't be there.

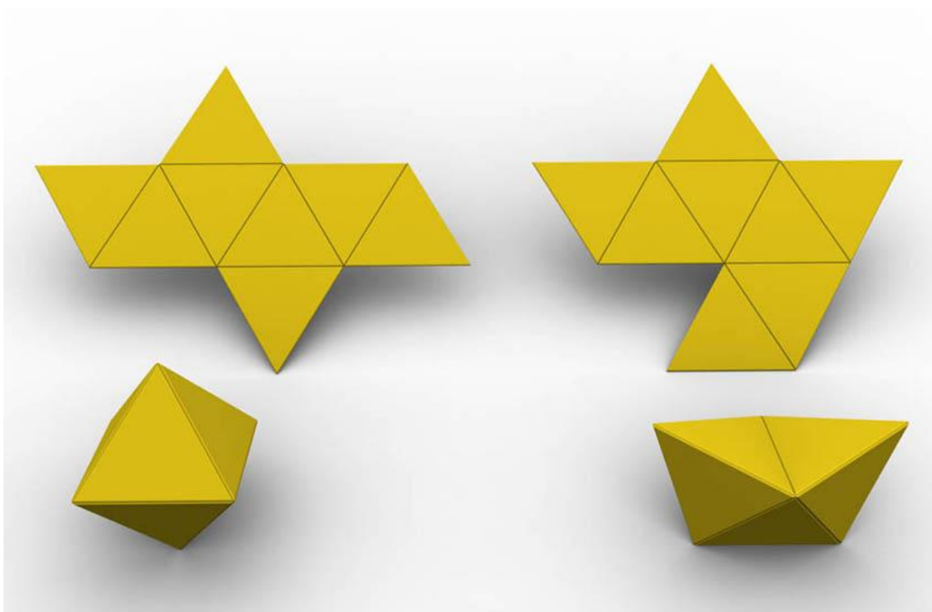

Figure 28: *The octahedron and the* "*boat* ".

10. References

1 Luca Pacioli – La divina proporcion – Ediciones Akal, S.A., Madrid, 1991

2 J.V. Field – Rediscovering the Archimedean Polyhedra. – Department of History of Art, University of London, 1996

3 Wilma di Palma – Polyhedra – I Poliedri della donazione Adriano Graziotti – Argos Edizioni, Roma, 1994