Connected Holes

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Abstract

It is possible to make interwoven structures by using two or more surfaces with holes. Several solutions are known. And also interwoven structures have been made with two or more 3D objects with holes. One example is M.C. Escher's 'Double Planetoid' [1], a combination of two tetrahedra. In this paper I show that we don't need two or more separate surfaces: even with one single surface we can make interesting interwoven structures which seem to be multi-layered, in 2D as well as in 3D.

1. Introduction

1.1. Interwoven surfaces. In Figure 1 an example of a structure consisting of two interwoven surfaces is shown. In both layers holes are made at such places that the surfaces can be woven. In all three interwoven layer structures of Figures 1, 2 and 3 the midpoints of the holes are placed on a square grid. The shapes of the holes in the layers are respectively rounded, elliptical and hexagonal. In addition to the shape of the holes, the weaving in the three examples is also different. The design of the interwoven structure in Figure 3 is made by M.C. Escher [2].

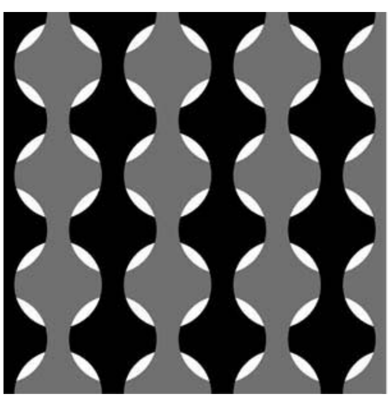

Figure 1: *Interwoven surfaces - a.*

Figure 2: *Interwoven surfaces - b.*

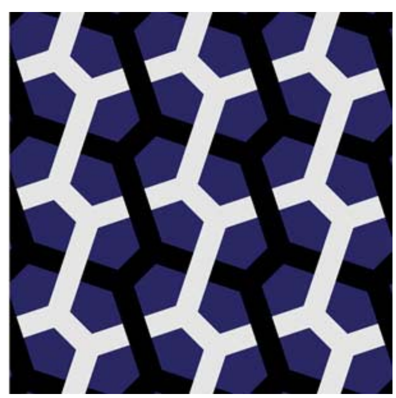

Figure 3: *Interwoven surfaces - c.*

1.2. Hexagonal grid. We can use almost any pattern as an underlying grid for an interwoven layer structure. A hexagonal grid is used in the examples of Figure 4 and Figure 5. Also the number of layers may vary. A structure of three interwoven layers is shown in Figure 5.

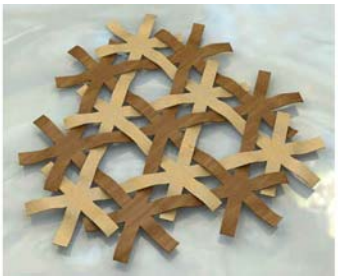

Figure 4: *2 Interwoven surfaces.*

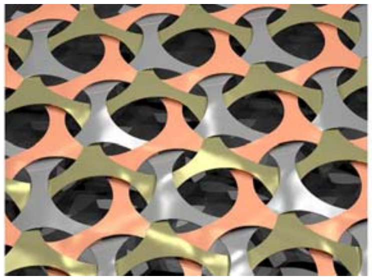

Figure 5: *3 Interwoven surfaces.*

1.3. 3D Structures. Besides the interwoven 2D layer structures there are also structures with interwoven 3D objects. As an example we can take M.C. Escher's woodcut 'Double Planetoid'. "Two tetrahedra going through each other, gliding through space as a planetoid. Together both objects are one connected structure, but they do not know of each others' existence", Escher said about this print [1]. And we can also say this about the two Möbius bands in Figure 6, or the two spheres in Figure 7.

Figure 6: *Entwined Möbius bands.*

Figure 7: *Entwined spheres.*

2. Connected Holes

2.1. Connected Rings. When we look closely at the structure in Figure 1 we see that this structure can be described in another way: it can be seen as a set of connected rings, as in Figure 8a. And in Figure 8b we can recognise the connected holes of the structure in Figure 2. So there seems to be a close relationship between both interpretations. A structure of interwoven layers can be translated into a structure of connected holes.

We may ask whether this translation step can be used in both directions. That is, when we start with a structure of connected rings, can this structure be translated into a structure of interwoven layers? And what will happen when we start with a structure in which a more complicated ring, like the knot in Figure 8c, is used?

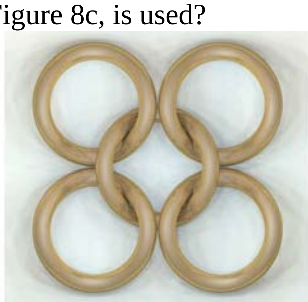

Figure 8a: *Connected rings.*

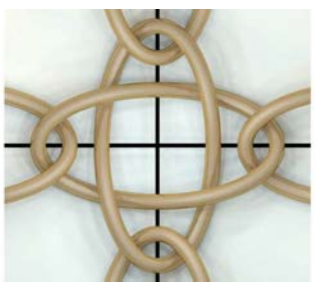

Figure 8b: *Connected ellipses.*

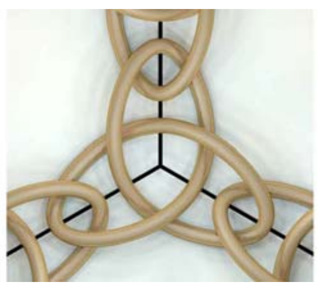

Figure 8c: *Connected knots.*

2.2. Connected knots. Starting with a trefoil knot (Figure 9a) we first connect this knot to three other trefoil knots (Figure 9b). When we continue this, we can create a pattern of connected trefoil knots (Figure 9c). The question now is: can we translate this pattern of connected knots into an interwoven layer pattern? To examine this we can use the soap film method. In Figure 9d you can see a surface that connects parts of the lines of the trefoils like a soap film. We can continue adding more surfaces in this way (see Figure 9e and following). After the third step (Figure 9f) we do not get a direct connection between the first and the last part of the surface. Only when we go around twice (Figures 9g, h, i) do we get the situation that both ends meet. And now it looks like a double layer structure but it still is one single surface. So in fact we have gotten an interwoven structure built with one single surface (Figure 10).

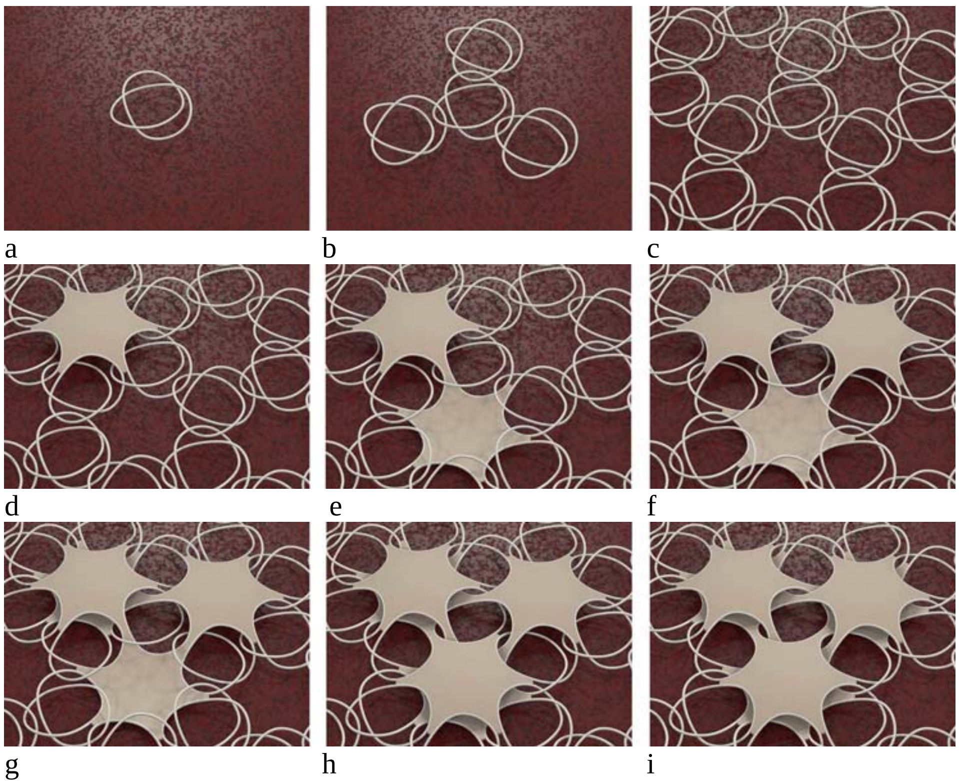

Figure 9: *Soap film on the knot structure.*

Figure 10: *Interwoven structure - hexagonal.*

2.3. Fivefold Knot. In the next example we start with the structure of Figure 3, M.C. Escher's design of two interwoven grids (Figure 11a). The first step is to replace the outlines of the blue pentagonal shapes, which are the holes of the structure, by fivefold knots. We do this in such a way that the fivefold knots become connected (Figure 11b). Also in this example we continue to add a soap film (Figures 11 c-h). And again we see that the result is a single surface interwoven layer structure that looks like a structure consisting of two different layers (Figure 12).

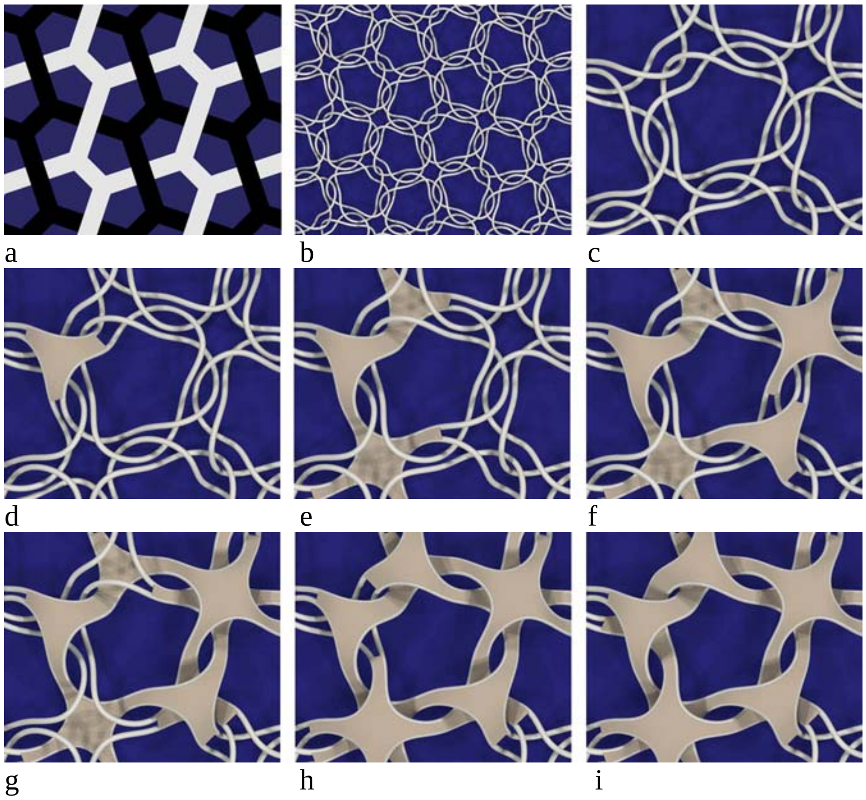

Figure 11: *Soap film on a net of fivefold knots.*

Figure 12: *Interwoven layer structure based on a fivefold knot pattern.*

3. Interwoven Surfaces.

3.1. Regular and semiregular tilings. The underlying pattern of the structure in Figure 10 is the regular tiling 6,6,6. The structure in Figure 12 has the semiregular (or Archimedean) tiling 4,3,4,3,3 as the underlying pattern. These tilings are composed of regular polygons such that at the vertices of each tile you will find the same configuration of regular polygons. And therefore the same number of tiles meet at each vertex. For the structure in Figure 10 this number is three, which is an odd number. And because this is an odd number we will get a knot figure as a hole in our interwoven layer structure (see Figure 8c). When this number is even we will get a combination of two or more rings (see Figure 8b).

At each vertex of the underlying pattern of the structure in Figure 12 we have 5 polygons (going around a vertex we see a square, a triangle, a square, and two triangles). Also in this tiling the number of polygons in each vertex is odd and so the resulting interwoven layer structure will again be a single surface, because of the knot-shaped holes. Other such surfaces are shown in Figures 13, 14, and 15.

Figure 13: *Tiling 6,6,6.*

Figure 14: *Tiling 8,8,4* .

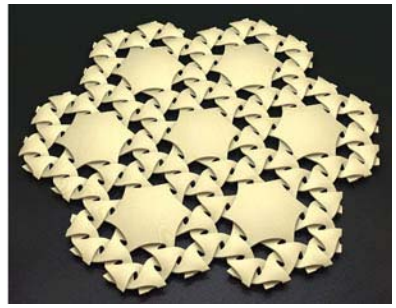

Figure 15: *Tiling 6,3,3,3,3.*

3.2. Dual grids. Starting with the Archimedean tilings we can generate another group of tilings, the so-called 'dual tilings'. When we connect the midpoint of each tile with the midpoints of its surrounding tiles we get a new pattern, the dual pattern of the tiling. In the dual tiling, all tiles are congruent. In Figure 16 this is demonstrated for the Archimedean tiling 4,3,4,3,3. Figures 17 and 18 show the interwoven structure and its dual. Figures 19-21 show additional dual interwoven structures. And again when odd numbers occur as in the number of tiles in a vertex of the tiling we will get interwoven structures built out of one single surface.

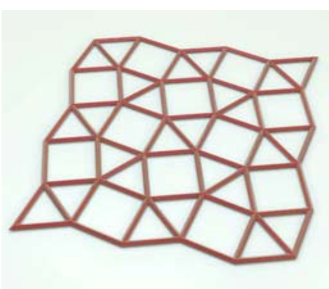

Figure 16a: *Tiling 4,3,4,3,3.*

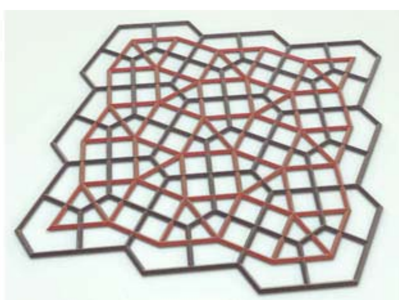

Figure 16b: *Creating the dual pattern.*

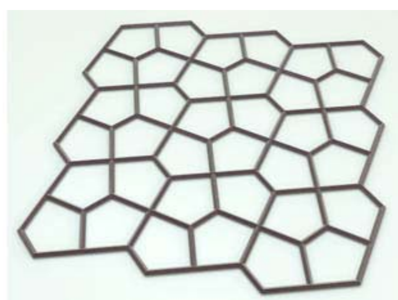

Figure 16c: *Dual of 4,3,4,3,3.*

Figure 17: *Base: tiling 4,3,4,3,3.*

Figure 18: *Base: dual of 4,3,4,3,3.*

Figure 19: *Base: dual of 6,3,3,3,3.*

Figure 20: *Base: dual of 6,3,6,3.*

Figure 21: *Base: dual of 6,4,3,4.*

4. 3D objects from 2D structures.

4.1. The Platonic Solids. The next step is to apply the concept of knot-shaped holes on 3D objects. There are five Platonic solids. Of these, the tetrahedron, the cube and the dodecahedron have in common the fact that at each vertex three faces meet. We can place a trefoil knot on these vertices as in Figure 22.

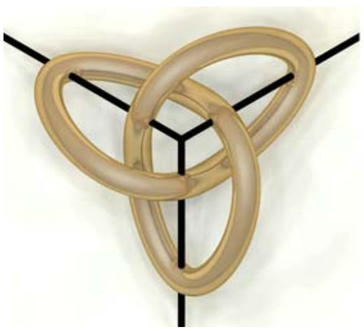

Figure 22: *Knot placed on a vertex of a solid.*

Figure 23: *Base: tetrahedron*

Figure 24: *Base: cube.*

On each of the faces of the solid we will get a net of connected knots like we had in the planar structures in section 2. And after adding the soap film surfaces we get the double layer objects shown in Figures 23, 24 and 25.

Figure 25: *Base: dodecahedron.*

Figure 26: *Base: snub cube.*

4.2. The Archimedean Solids. Also the Archimedean solids can be used as a basic structure for a three-dimensional interwoven single surface object. For the example shown in Figure 26, the snub cube is used as the basic structure. The object is built out of a single continuous surface.

5. M.C. Escher Stars

5.1. M.C. Escher's 'Gravity'. In the print 'Gravity' M.C. Escher has used a star-shaped object on which twelve animals are situated [1]. For each animal there is a floor to stand on and a roof to cover. The floor and the roof can be seen as two layers. But the complete object is not a combination of two objects as in M.C. Escher's print 'Double Planetoid' [1]. In this object (Figure 27) the star-shaped planes are connected in such a way that we can walk from any plane to any other plane.

5.2. Interwoven Layer in 3D. The star-shaped object M.C. Escher has used in his print is one continuous surface, but we can see clearly that the object has two layers. This is in fact the same situation as we have seen in the interwoven layer structures in section 3. To make this clearer, we can enlarge the holes of M.C. Escher's object as in Figure 28. And when we zoom in to one of the holes (Figure 29) we recognize the same trefoil knot we have used in the objects shown in paragraph 4.1. In fact after rounding the tops of the pyramids of M.C. Escher's object we will get the same object as shown in Figure 25.

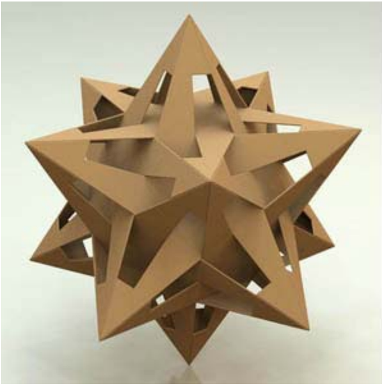

Figure 27: *M.C. Escher's object.*

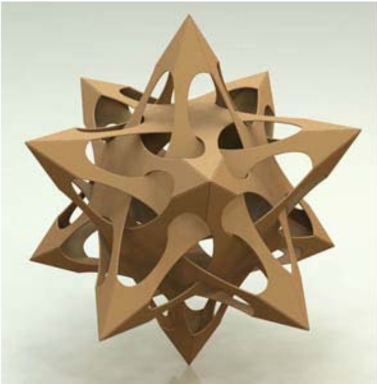

Figure 28: *Enlarged holes.*

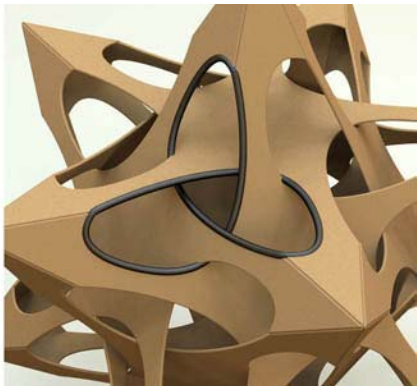

Figure 29: *Trefoil knot.*

5.3. Stellation. Star-shaped objects can be created starting from a Platonic or an Archimedean Solid. As an example, this process is shown in Figure 30 where we start with the icosahedron (Figure 30a). In the first three steps (Figures 30b, 30c, and 30d) we extend the triangular faces of the icosahedron until they touch each other again. In Figure 30d this process is completed and the result is a star polyhedron. Note that this object has two layers: beneath each pyramid there is still the original triangular face. We will make this visible by cutting holes in the pyramids (Figures 30e and 30f). And in the final steps (Figures 30g, 30h, and 30i) we enlarge the holes so that the shape of the hole, which is in this case a fivefold knot, can be seen clearly. The result is an interwoven structure built with one continuous surface (Figure 31).

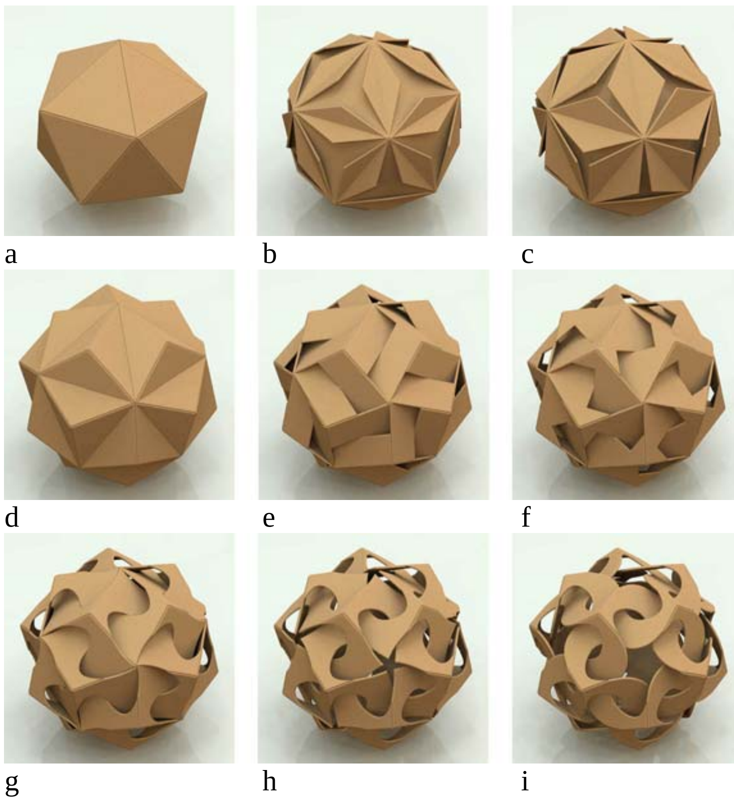

Figure 30: *Stellation of the icosahedron.*

Figure 31: *Double layer structure based on the icosahedron.*

6. Interwoven Surfaces in 3D

6.1. Two layers. The stellation process can be applied not only to the Platonic and Archimedean solids, but also to the duals of these solids. In some cases we will end up with a combination of two objects— for example, starting with the octahedron the result of the stellation process will be a combination of two tetrahedra as in M.C. Escher's "Double Planetoid" [1]. In other cases it will lead to one complete object. The outcome depends on the number of faces that meet at each vertex. When this number is even as in the octahedron (4 triangles in every vertex) then the result will be a combination of two objects. When this number is odd the result will be one double-layered object. The following examples are derived from the rhombic dodecahedron (Figures 32 and 35), the dual of the snub cube (Figures 33 and 36) and from the rhombic triacontahedron (Figures 34 and 37).

Figure 32: *Base:rhombic dodecahedron.*

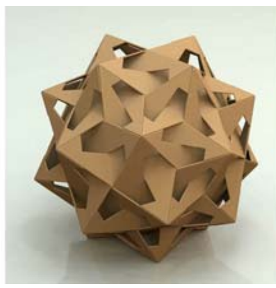

Figure 33: *Base snub cube.*

Figure 34: *Base:rhombic triacontahedron*

Figure 35: *Base:rhombic dodecahedron.*

Figure 36: *Base snub cube.*

Figure 37: *Base:rhombic triacontahedron*

We can make three different stellations that are made of sixty identical faces (Figures 38, 39 and 40). The object shown in Figure 40 and Figure 41 are both based on the dual of the snub dodecahedron.

Figure 38: *Star: 60 faces - a.*

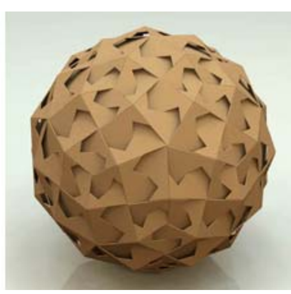

Figure 39: *Star: 60 faces - b.*

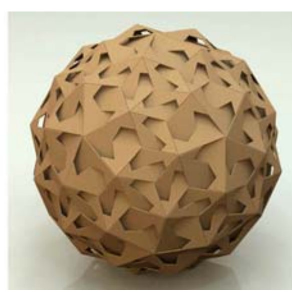

Figure 40: *Star: 60 faces - c.*

Figure 41: *Base: dual of the snub dodecahedron*

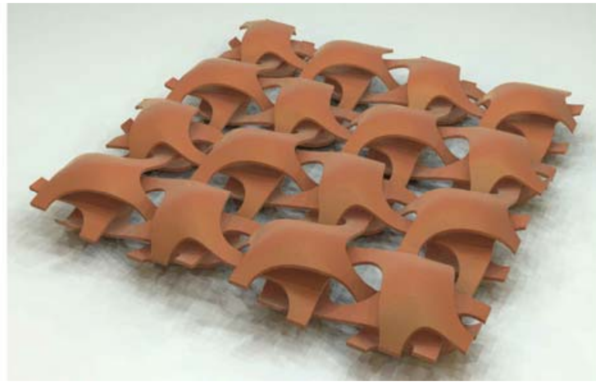

Figure 42: *3 layers – 1 surface.*

7. More Layers

7.1. Three-layer Structures. By applying connected knots we were able to create double layer structures. Knots are related to star polygons. The fivefold knot is derived directly from the five pointed star, a pentagon of order 2. With bigger polygons, like for example the octagon, higher orders also occur. The structure shown in Figure 42 is made by applying the star octagon which has order three. The result is a three-layer interwoven structure built with one continuous surface. There is a nice way to create three-layer structures in 3D: we can use the interwoven layer structure of Figure 43, which has three separate layers, as 'material' to make a folding pattern for an icosahedron. The result will be the three-layered single surface object shown in Figure 44.

Figure 42: *3 layers – 3 surfaces.*

Figure 44: *3 layers – 1 continuous surface.*

References

[1] M.C. Escher, *Grafiek en Tekeningen*, Uitgeverij Tijl, Zwolle, 1960. (*The Graphic Work of M.C. Escher*, Meredith Press, New York, 1960; Ballantine Books, 1971; Wings Books, New York, 1996.)

[2] Doris Schattschneider, *M.C. Escher: Visions of Symmetry*, Abrams, 2004.